

# R&D and Markets for New Knowledge over Firms' Life-cycles\*

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March 2007

## Abstract

This paper develops a model of R&D with firms' life-cycles. When a young firm creates new knowledge, it is hard to predict if the firm will be able to implement the knowledge successfully. Nevertheless, once the firm implements the knowledge successfully (and becomes established), this success signals that the firm will be able to repeat such a success by implementing another knowledge of the same type in the future. Studying a dynamic equilibrium of the economy populated by both young and established firms, we derive the implications on the market for knowledge. We find that established firms are more likely to buy knowledge than young firms because established firms can, but not young firms, selectively buy the knowledge that the firms will be able to implement successfully. Our model stresses that the questions who innovates (creates knowledge) and who introduces innovations to the market (implements knowledge) should be distinguished and the answers to both questions depend on the transaction cost of knowledge transfer. To be concrete, we find that as the transaction cost becomes lower, incumbents innovate less relative to entrants, whereas incumbents introduce more innovations relative to entrants.

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\*We are grateful to Andrea Caggese, Bruno Cassiman, Andrea Fosfuri, Uli Hege, Boyan Jovanovic, Pierre Mella-Barral, Anne Miner, Merih Selvir and seminar participants at Wisconsin, Universitat Pompeu Fabra, HEC Lausanne, 2006 RICAFAE conference, University of Mannheim, Universitet van Amsterdam, HEC Paris and Kyoto University for helpful comments on this project. Usual disclaimers apply.

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## Abstract

This paper develops a model of R&D with firms' life-cycles. When a young firm creates new knowledge, it is hard to predict if the firm will be able to implement the knowledge successfully. Nevertheless, once the firm implements the knowledge successfully (and becomes established), this success signals that the firm will be able to repeat such a success by implementing another knowledge of the same type in the future. Studying a dynamic equilibrium of the economy populated by both young and established firms, we derive the implications on the market for knowledge. We find that established firms are more likely to buy knowledge than young firms because established firms can, but not young firms, selectively buy the knowledge that the firms will be able to implement successfully. Our model stresses that the questions who innovates (creates knowledge) and who introduces innovations to the market (implements knowledge) should be distinguished and the answers to both questions depend on the transaction cost of knowledge transfer. To be concrete, we find that as the transaction cost becomes lower, incumbents innovate less relative to entrants, whereas incumbents introduce more innovations relative to entrants.

# 1 Introduction

Acquiring knowledge from external sources have recently become routine activities for many established firms.<sup>1</sup> For instance, Merck spent about \$200 MM on third-party collaborations between 1998 and 2000 and, by 2000, one-third of Merck's products came from external research or licensing collaborations (Pisano, 2002). Grindley and Teece (1997) point to the increasing use of technology licensing by companies such as IBM, Hewlett-Packard, Texas Instruments, and AT&T during the 1990s. Although comprehensive evidence does not exist, young firms appear to be playing an important role in supplying knowledge to established firms. For instance, since 1993, Cisco acquired many young companies which had invented new types of network equipment. (Eisenhardt and Sull, 2001) Bristol-Myers bought the exclusive manufacturing and marketing right of the cancer drug, Erbitux from Imclone which was founded in mid 80s. Hall and Ziedonis (2001) report the strong presence of young firms that design semiconductors for manufacturers.

Despite the growing importance of the market for knowledge in which established and young firms actively interact, few papers provide a theoretical framework to understand this market. This paper departs from the existing literature on the market for knowledge by studying *equilibrium of the market* for knowledge and the role of firms' life-cycles in the formation of the market for knowledge. Our model addresses issues such as under which condition does a firm specialize in development instead of perform both research and development?; which firm becomes a buyer/seller of knowledge?; what is the equilibrium price for knowledge?

To address these issues, we develop a model of the innovation process through which a firm creates knowledge (research) and the same or another firm implements the knowledge into new goods or services (development). Implementation of knowledge generates profit only if the firm that implements has the right capability. Different types of knowledge require different types of capability and therefore the firm that can implements one type of knowledge may not possess the capability to implement another type of

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<sup>1</sup>Arora, Fosfuri and Gambardella (2001) gives an excellent survey of this issue.

knowledge.

We distinguish young firms from established firms as follows. A young firm does not know which type of knowledge it can implement. Nevertheless, the first time the firm implements the knowledge successfully, the firm learns that it possesses the capability to implement the knowledge of the same type and becomes “established.” Because the established firm can, but the young firm cannot, identify the knowledge that it can implement, the established firm is more likely to buy knowledge from other firms than the young firm.<sup>2</sup>

Studying stationary equilibrium with a continuum of firms, we find that three types of equilibrium exist depending on the transaction cost of selling knowledge. When the transaction cost is low, young firms sell their knowledge to established firms. When the transaction cost is moderate, established firms trade their knowledge each other. When the transaction cost is high, the market for knowledge does not exist.

When the transaction cost is low, established firms specialize in implementing knowledge bought from young firms. Because established firms know but young firms do not know which type of knowledge they can implement, established firms have the advantages in implementing knowledge. As a result, their specialization in implementing knowledge is efficient and naturally arises as equilibrium when the transaction cost does not impede the reallocation of knowledge created by young firms to the established firm. Some knowledge created by young firms remain with and implemented by the young firms, as the implementation capacity of established firms is limited. We call this equilibrium exploitation equilibrium because established firms specialize in exploiting knowledge created by young firms and never explore.<sup>3</sup>

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<sup>2</sup>Similarly to our model, Gilbert and Newborn (1982) also contend that incumbents have higher willingness to pay for innovations. Nevertheless, their logic is based on competition; Monopoly profit is higher than duopoly profit and therefore the monopolist’s incentive to maintain its monopoly status should be bigger than the new firm’s incentive to enter the market. Supporting that incumbents (established firms in our paper) have higher willingness to pay for innovations, Carnitve and Kraft (2005) find that incumbents show higher expenditures for technology licenses than potential entrants.

<sup>3</sup>Cassiman and Ueda (2006) argue that an established firm often forego the opportunities to implement promising innovations due to their low fit to the firm’s capability and

In exploitation equilibrium, we find that two factors influence the ratio of knowledge that young firms sell to established firms against knowledge that young firms attempt to implement by themselves. First, when technological innovation is rapid and new knowledge obsoletes quickly, established firms have to come back to the market for knowledge more often and therefore more knowledge is bought and sold instead of implemented by the young firms. Second, interestingly, when young firms attempt to implement the knowledge is more likely to fail, they are more likely to make such an attempt instead of selling their knowledge to established firms. This paradox arises because when such an implementation attempt is likely to fail, only few young firms become established firms. As a consequence, less established firms exist in a stationary equilibrium and aggregate demand for knowledge is low. As a result, more young firms are forced to attempt to implement their knowledge instead of selling it to established firms.

When the transaction cost is moderate, not only young firms but also established firms create new knowledge. In this equilibrium, the transaction cost does impede the reallocation of knowledge created by young firms to established firms. Consequently, young firms attempt to implement their knowledge instead of selling it to established firms. Because young firms may not have the right capability for the knowledge they attempt to implement, such attempts often fail. In this equilibrium, the market for knowledge still exists between established firms. The price of knowledge is too low to motivate young firms to sell their knowledge, but still high enough to motivate established firms to sell their knowledge that do not match the firms' implementation capabilities.

When the transaction cost is high, the market for knowledge is closed. Every firm both creates and implements knowledge it creates. In this equilibrium, established firms shelve many of their knowledge due to their mismatch with the firms' implementation capabilities.<sup>4</sup> Differently from es-

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to its limited implementation capacity.

<sup>4</sup>For example, according to Ramie (2007), Proctor & Gamble, which uses only some 7,000 patents of its approximately 36,000, patents any meaningful advance but only acts upon those that are aligned with P&G's long-term strategy. Company spokesman, Jeff Leery said "in some cases we have a technology that for whatever reason we decide we're

established firms, young firms do not know which type of knowledge they can implement and therefore are less selective than established firms in implementing knowledge. This implication of the model is consistent with the finding by Guedj and Scharfstein (2006) on clinical trials. They find that younger firms are more likely to proceed to the second stage of the clinical trial, but they are more likely to fail.

We can interpret the transaction cost of technology transfer in two ways. First, the transaction cost is lower if intellectual property rights to innovations are more strongly protected because inventors do not need to build less means of appropriating the returns to innovations other than patenting their inventions. Second, the transaction cost is lower if sellers can find buyers easily.<sup>5</sup> U.S. pro-patent regime since early 1980s and technological innovation in communication technology exemplified by the emergence of internet presumably both reduced the transaction cost of selling technology. As a result, our model predicts that the market for technology should have developed recently. If the transaction cost becomes low enough to sustain exploitation equilibrium, then our model predicts that established firms buy knowledge discovered by young firms and specialize in implementing knowledge. Consistent with this prediction, Hunt and Nakamura (2007) find that R&D intensity of younger firms increased much faster than that of established firms in the last few decades.

Our model provides a new perspective to the debate on innovativeness of incumbent versus entrant. Since Gilbert and Newbery (1983) and Reinganum (1983), economists have long debated which incumbent or entrant innovate more. Evidence on this issue is mixed. For example, Lerner (1997) finds that entrants are more innovative than incumbents in the U.S. hard disk drive industry, whereas Blundell, Griffith and van Reenen (1999) and Schroth and Szalay (2006) find the opposite in the panel of British manufacturing industries and in the U.S. pharmaceutical industry, respectively.<sup>6</sup>

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not going to launch, or it needs more development beyond P&G's expertise."

<sup>5</sup>Hellmann (2006) argues that a patent system also helps sellers of innovations to find their buyers, through a centralized invention depository system.

<sup>6</sup>Related with this debate, some literatures report that young firms appear to be more innovative than established firms are (Scherer, 1980). Small firms, who are often young,

Our model stresses that the two questions, who innovates (creates knowledge) and who introduces innovations to the market (implements knowledge), should be distinguished and the answers to both questions depend on the transaction cost of knowledge transfer. To be concrete, our model implies that as the transaction cost becomes lower, incumbents innovate less relative to entrants, whereas incumbents introduce more innovations relative to entrants.

Our paper is related to several strands of literature. Jovanovic (1981) is a seminal article that studies the implication of the adaptive learning on the firm's exit and growth decision over its life-cycle. In his model, a young firm is yet to learn its potential profitability, whereas an established firm has already learned its own profitability through its past performance. As a consequence, the young firm is more likely to learn from and respond to its recent performance than the established firm. Nevertheless, knowledge are homogeneous in the Jovanovic model, and therefore it does not address the issue of project selection that is the focus of our paper. Similarly to our paper, Cassiman and Ueda (2004) model the difference between a new firm and an established firm such that a new firm is indifferent between knowledge, whereas an established firm is not. Nevertheless, differently from this paper, Cassiman and Ueda do not study the process through which a new firm becomes an established firm. Similarly to our paper, Holmes and Schmitz (1990) model a multi-stage project and study individuals that possess the same commercialization skill but differ in the incubation skill. In their model, knowledge are transferred because some individuals are more talented in starting up new knowledge than others, whereas, in our model, knowledge are transferred because incubators may not possess the right capability to develop the knowledge. Holmes and Schmitz, and this paper share the same prediction such that transferred knowledge are more profitable than non-transferred knowledge. Differently from Holmes and Schmitz, we are also able to predict that a business is transferred from a young firm to

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tend to do more research than implementation (NSF, various years publication). Small firms appear to produce more innovations given the same number of employees (Acs and Audretsch, 1988).

an established firm. Similar to our paper, Bernardo and Chowdhry (2000) study the project selection problem when a firm learns its capability by experimentation. They assume that a firm may possess either project-specific capability or general-management capability. In equilibrium, an established firm diversifies into a new area to see which type of capability the firm possesses. If the diversification is successful, the firm learns that it possesses the general-management capability. Otherwise, the firm learns that it possesses project-specific capability. Differently from our paper, the focus of Bernardo and Chowdhry is the firms that have once successfully implemented their past knowledge. As a result, their model is more appropriate for explaining the learning of established firms rather than that of young firms, which is the focus of our paper. Finally, Mitchell (2000) studies the model in which a firm acquires a project specific capability through experience. In our model, the firm creates instead of acquires such a capability.

The organization of this paper is as follows. Section 2 describes the basic model in which a firm cannot sell the knowledge it creates to another firm. Section 3 introduces the market for knowledge and derives several testable implications. For simplicity, Section 2 and Section 3 both assume that a young firm exits if it fails in implementing knowledge. Differently from Section 2 and Section 3, Section 4 extends the model such that a young firm may stay in the economy after it fails in implementing knowledge.

## 2 The Model

We outline the life-cycle of the risk-neutral firm, which maximizes the expected present value of cash flows at the continuously compounded discount rate  $r$ . Firm's capability is denoted by  $s_f \in T_S$ , where  $T_S = \{1, 2, \dots, S\}$ . This capability defines which type of knowledge the firm can implement profitably. The capability of the firm does not change over time. Knowledge's type has the same space as  $T_S$ , and the firm can implement knowledge profitably if and only if the firm's capability and the knowledge's type are the same. We call the firm "young" if it does not yet know its own capability and "established" if it learned its own capability. We assume that the prior

probability that a young firm possesses capability  $s_f$  is  $1/S, \forall s_f \in T_S$ .<sup>7</sup>

In this section, we do not allow a firm to sell its knowledge to another firm and focus on describing basic differences between young and established firms. This setup corresponds to the case in which the transaction cost of selling knowledge is too high. In the next section, we introduce a possibility for a firm to sell its knowledge to another firm and show that the market for knowledge exists if the transaction cost is sufficiently low.

## 2.1 Project Arrival, Incubation, and Learning

The firm's life-cycle develops as follows. The firm emerges as a young firm and may incur per period cost,  $c_0$  to create knowledge. The knowledge creation occurs according to a negative exponential arrival process with rate of  $\lambda_0$ . The prior probability that created knowledge has type  $s_p$  is  $1/S, \forall s_p \in T_S$ . The firm may also suddenly die according to a Poisson process with death rate,  $\mu_g > 0$ . This probability is invariant across firm's different stages. If we denote  $C_{g/0}$  be the expected present value of the total creation cost, then  $C_{g/0} = c_0 / (r + \lambda_0 + \mu_g)$ . Once the young firm creates knowledge, it decides whether to implement or to abandon the knowledge. If the knowledge is implemented, it costs  $c_1$  per period until the implementation outcome is revealed. This revelation timing follows a negative exponential arrival process with rate  $\lambda_1$ . If we denote  $C_{g/1}$  as the present value of the expected implementation cost,  $C_{g/1} = c_1 / (r + \lambda_1 + \mu_g)$ . The outcome of the implementation is either success or failure. If it is successful, the firm starts generating a per-period profit  $y > 0$  for an uncertain period if  $s_p = s_f$ , and otherwise the firm receives no return on its implementation effort.<sup>8</sup> We interpret  $y$  as the net profit per unit time a firm enjoys from a successful

<sup>7</sup>We can generalize the results of our model to the case that a firm may have no capability to implement knowledge. For this generalization, it suffices to include another firm type, say 0, in the firm's type space.

<sup>8</sup>A more general setting is to assume that  $y$  is a decreasing function of the distance between  $s_p$  and  $s_f$ . This generalization, however, adds the complex learning problem of a young firm. For instance, after generating a sufficiently high revenue, the young firm tries to implement if and only if the new knowledge type is sufficiently close to that of the previous knowledge. Studying this learning and experimentation process of the young firm is out of this paper's scope.

knowledge as long as it acts as a monopolist. However, after some time, the market becomes competitive driving profits to zero. If a young firm starts generating revenue from knowledge of type  $s_p$ , it learns that its capability is equal to  $s_p$  and therefore becomes established. For simplicity, we assume that the young firm exits if the implemented knowledge flops. One rationale for this assumption is financing frictions such as moral hazard. In the presence of such frictions, a firm may find it difficult to raise funds from external sources and therefore has to finance their investment internally. Failing in implementing one project exhausts the internal cash of the firm and, as a result, the young firm needs to exit if it fails in implementing the first project.

It is useful to discuss two issues on our basic setup at this moment. Second, “firm” characterized by  $s_f$  may be a collection of resources such as entrepreneur, partners, knowledge, physical assets including location, or unique products or services. Our “firm” is a distinct entity from its knowledge.

Similarly to a young firm, an established firm may also die according to a Poisson process with death rate  $\mu_m > 0$ . A positive profit stream of the established firm stops with the Poisson death rate  $\lambda_2$ . To maintain the model as simple as possible, we assume that  $\mu_m = \mu_g = \mu > 0$ . Therefore, if we denote  $Y$  be the expected present value of the revenue stream from the knowledge, then  $Y = y/(r + \lambda_2 + \mu)$ . Once the knowledge “dies” in this way, the established firm enters knowledge process similar to the one for a young firm. The established firm first creates a new knowledge, and, once created, decides to implement it. If the established firm decides not to implement the knowledge, it waits for another knowledge to arrive. The flow cost and the arrival and death rates are the same as described in the case of a young firm.

We denote the stage of firm’s development by  $\sigma \in \{g/0, g/1, m/0, m/1, m/2\}$ , where  $g$  means that a firm is young and  $m$  means that a firm is established. The symbol  $/0$  means that the firm is engaged in creating knowledge,  $/1$  means that the firm is implementing knowledge, and  $/2$  means that the firm is generating revenues. These assumptions are summarized in the following

	Young Firm		Established Firm		
	Creation	Implementation	Harvesting	Creation	...
	0	1	2	0	...
Transition Rate	$\lambda_0$	$\lambda_1$	$\lambda_2$	$\lambda_0$	...
Death Rate	$\mu$	$\mu$	$\mu$	$\mu$	...
Cash Flow Rate	$-c_0$	$-c_1$	$y$	$-c_0$	...

Table 1: Summary of Notations

table.

It is convenient to work with discount factors instead of discount rates, and therefore we introduce the following notation:

$$\rho_j = \frac{\lambda_j}{\lambda_j + r + \mu}, \quad j = 0, 1, 2.$$

To make the model non-trivial, we make the following assumption.

**Assumption 1**

$$\Phi - \rho_0 \frac{(S-1)C_1}{S} + \rho_0 \rho_1 \rho_2 \frac{\alpha}{S} \Phi \geq 0, \quad (1)$$

where

$$\Phi \equiv \rho_0 \rho_1 \frac{Y}{S} - C_0 - \rho_0 \frac{C_1}{S}, \quad \text{and} \quad (2)$$

$$\alpha \equiv \left[ 1 - \frac{\rho_0 \rho_1 \rho_2}{S} - \frac{\rho_0 (S-1)}{S} \right]^{-1}. \quad (3)$$

Intuitively,  $\Phi$  is the present value of the expected cash flows from the entire cycle of one knowledge for a *established* firm. The value  $\Phi$  includes the creation cost, the implementation cost, and the revenues if the knowledge is matched with the firm's capability, but does not include the implementation cost if the knowledge does not turn out to match the firm's capability. The left hand side of the equation (1) is the net present value of a young firm at its inception. The first two terms are the expected present value of the first knowledge. Note that we subtract the extra term  $\rho_0 (S-1) C_1/S$  because a young firm does not know its capability and always implements the

knowledge. The third term is the option value of becoming an established firm in the future. The variable  $\alpha$  is the multiplier to summarize the value of the knowledge that a established firm will receive eternally. In sum, this assumption ensures that initiating the process to discover knowledge has a non-negative net present value for a young firm.

## 2.2 Implementation Policy and Value of the Firm

We now describe the implementation policy of the firm once the knowledge is created. Here, the decision may depend on whether the firm is young or established. The established firm knows its capability, therefore it knows with certainty if the knowledge will or will not generate revenues once it is implemented. The young firm, on the contrary, does not possess such prior knowledge and therefore the revenues that will be generated from the knowledge are uncertain.

**Definition 1** *History is a function  $h_\tau : [0, \tau] \rightarrow \sigma$ .*

That is, the history tracks the firm's past stage of its life-cycle.

**Definition 2** *A policy is a set-valued correspondence  $d : \{2^{T_S}, h_t\} \rightarrow \{0, 1\}$ , where  $T_S = \{1, 2, \dots, S\}$ ,  $2^{T_S}$  is the power set of  $T_S$ , that is, the set of all subsets of  $T_S$ . The decision  $d(\cdot) = 1$  if the firm decides to implement and  $d(\cdot) = 0$  otherwise.*

For notational simplicity, we will write  $d(s)$  instead of  $d(\{s\})$  from now on.

We now define the value of the firm for each stage of the firm's life-cycle. Let  $i = \{g, m\}$  be the firm's "age." That is, if  $i = g$ , the firm is young and if  $i = m$ , the firm is established. Let  $j = \{0, 1, 2\}$  be the knowledge-related stage of the firm. That is, if  $j = 0$ , the firm is attempting to discover knowledge, if  $j = 1$ , the firm is implementing knowledge, and if  $j = 2$ , the firm is generating a positive revenue. Using this notation, let  $V_{i/j}(d)$  be the value of the firm that follows policy  $d$  when the firm's age is  $i$  and the firm's knowledge-related stage is  $j$ . Then we define optimality as follows:

**Definition 3** A policy  $d^* \in 2^{Ts}$  is optimal for a firm if and only if  $V_{i/j}(d^*) \geq V_{i/j}(d), \forall d \in 2^{Ts}$ .

We introduce some terminology about firm's decisions that will be useful in the equilibrium analysis that follows.

**Definition 4** A policy  $d$  is called connected if for any  $s_1 < s_2$  with  $d(s_1, h_\tau) = d(s_2, h_\tau) = 1$  we have  $d(\tilde{s}, h_\tau) = 1, \forall \tilde{s} \in [s_1, s_2]$ . Policies that assign 1 for singletons will also be called connected by convention. A policy  $d$  is called monotonic if for any  $s_p \in T_S$  with  $d(s_p, h_\tau) = 1$  we have  $d(s, h_\tau) = 1$ , for all  $s$  such that  $s \geq s_p$ .

### 2.3 The Optimal Implementation Policy

We now characterize the optimal implementation policy of the firm. Recall that an established firm knows its capability  $s_f$ . Intuitively, it would only implement knowledge that match its capability, since this guarantees the firm to generate revenues. If the firm's capability does not match with the knowledge type, the firm will abandon the knowledge in order to avoid incurring the implementation cost  $c_2$ . The next proposition formalizes this intuition.

**Proposition 1** The optimal policy for an established firm of type  $s_f$ ,  $d^*$  is

$$d^*(s_p, h_\tau) = \begin{cases} 1, & \text{if } s_p = s_f \\ 0, & \text{otherwise} \end{cases}.$$

**Proof.** See Appendix.

The optimal policy of the established firm is history independent. The firm maximizes its value by following the same decision rule for all the knowledge it encounters during its lifetime.

The young firm, in contrast with the established firm, does not know its capability in advance. Therefore, there is uncertainty about if the implementation will be successful.

**Proposition 2** The optimal policy for the young firm is to implement the new knowledge it creates, independent of  $s_p$ .

**Proof.** See Appendix.

The proof of this proposition formalizes the intuition that the policy of implementing immediately brings  $V_{m/0}$  to the firm earlier than any other policy.

Proposition 1 and Proposition 2 make a sharp contrast between the established firm and the young firm in terms of their implementation policy. When making the implementation decision, the established firm is more selective than the young firm, which is more liberal about its choice of knowledge to implement. In what follows, for notational simplicity, we drop the dependence of the firm value on the policy  $d$  with the understanding that we refer to the optimal policies by all firms.

## 2.4 The Value of the Firm

We now derive the closed form solutions for the firm value.

**Proposition 3** *The firm value for each product stage is given by the following set of the equations:*

$$V_{m/0} = \alpha\Phi, \quad (4)$$

where  $\alpha$  and  $\Phi$  are given in Assumption 1,

$$V_{m/2} = Y + \rho_2 V_{m/0}, \quad (5)$$

$$V_{m/1} = -C_1 + \rho_1 V_{m/2}, \quad (6)$$

$$V_{g/1} = -C_1 + \frac{\rho_1 V_{m/2}}{S}, \quad (7)$$

and

$$V_{g/0} = -C_0 + \rho_1 V_{g/1} \quad (8)$$

**Proof.** See Appendix.

Note that the value of the firm is linear on the knowledge potential revenue  $Y$ , the creation cost  $C_0$  and the implementation cost  $C_1$ . The value of the firm is decreasing in the number of knowledge types  $S$ , because if  $S$  increases, then it becomes less likely that the young firm's capability matches

with the knowledge the firm creates. Observe that except  $V_0$ , which is nonnegative according to equation (1), all other values are strictly positive. Equation (1) implies  $\Phi > 0$  and  $\alpha > 0$ . Consequently  $V_{m/0} > 0$ , and it follows that  $V_{m/2} > 0$  by equation (5) and by  $Y > 0$ . To show that,  $V_{i/1} > 0$ ,  $i = \{g, m\}$ , it suffices to demonstrate that  $V_{g/1} > 0$ , because  $V_{m/1} > V_{g/1}$ . Sticking equation (5) for  $V_{m/2}$  into equation (7) gives

$$\begin{aligned} V_{g/1} &= -C_1 + \frac{\rho_1}{S}Y + \frac{\rho_1\rho_2}{S}V_{m/0} \\ &\geq -C_1 + \frac{\rho_1}{S}Y \\ &> 0. \end{aligned} \tag{9}$$

The last inequality follows equation (2) together with  $\Phi > 0$ . By using a similar logic, we can prove that  $V_{g/0} > 0$ .

#### 2.4.1 Value of Established Firm versus Value of Young Firm

We now compare the value of a young firm with that of an established firm for the same product stages. For the implementation stage, subtracting equation (7) from equation (6) and recursively using equation (5) and equation (6), we have

$$V_{m/1} - V_{g/1} = \rho_1 \frac{S-1}{S} (Y + \rho_2 \alpha \Phi). \tag{10}$$

This difference between the value of an established firm and that of a young firm at the implementation stage comes from the payoff difference when the knowledge implemented by the young firm turns out not to match with the young firm's capability. If this is the case, the young firm does not enjoy both the value of positive revenue stream  $Y$  and the continuation value after the end of the revenue stream  $\alpha\Phi$ , whereas the established firm enjoys both values.

Similarly for the creation stage, we can compute

$$V_{m/0} - V_{g/0} = \rho_0 \frac{S-1}{S} (C_1 + \alpha\Phi). \tag{11}$$

The right hand side is the sum of the cost saving and the continuation value that the established firm can potentially realize if the knowledge does not

match with the firm's capability. For any product life stage, the value of the established firm is at least as much as that of a young firm. This is because the established firm knows its capability and this information helps the firm to avoid incurring the implementation cost of the unmatched knowledge.

How do exogenous variables affect the value of young and established firms? The answers to this question is summarized in the following corollaries.

**Corollary 1** *The impact of the knowledge revenue and of its duration on the firm values is as follows. For  $j = \{0, 1\}$ ,  $\partial V_{m/j}/\partial y > \partial V_{g/j}/\partial y > 0$  and  $\partial V_{m/j}/\partial \lambda_2 < \partial V_{g/j}/\partial \lambda_2 < 0$ . For  $j = 2$ ,  $\partial V_{m/j}/\partial y = \partial V_{g/j}/\partial y > 0$  and  $\partial V_{m/j}/\partial \lambda_2 = \partial V_{g/j}/\partial \lambda_2 < 0$ .*

**Proof.** To prove the positive impact of  $y$ , note that  $\Phi$  increases in  $y$ . To prove the negative impact of  $\lambda_3$ , note that differentiating  $\alpha\Phi$  by  $\lambda_3$  gives

$$\frac{\partial \alpha\Phi}{\partial \lambda_2} = \frac{\alpha\rho_0\rho_1 r}{S(r + \lambda_2)^2} \left( \alpha\Phi - \frac{y}{r} \right) < 0.$$

Together with these two facts, partial differentiation of equations (5), (6), (7), (4), (8), (10), and (11) gives the results. ■

This corollary states that the value of the firm is increasing in the knowledge revenue  $y$  and the duration of this revenue  $1/\lambda_2$ . It also states that the sensitivity of the value to  $y$  and  $\lambda_2$  is higher for an established firm than for a young firm. This asymmetric sensitivity is due to the following reason. The established firm will stay in the market unless it involuntarily dies, whereas the young firm may exit the market if its knowledge turns out not to match its own capability. Change in  $y$  and  $\lambda_2$  affects the firm value only for the state in which the firm stays in the market. A young firm is affected less because it, unlike the established firm, has a lower probability to exit from the market.

**Corollary 2** *The impact of the creation cost and its duration is as follows:  $\partial V_{m/0}/\partial c_0 < \partial V_{y/0}/\partial c_0 < 0$  and  $\partial V_{m/0}/\partial \lambda_0 < \partial V_{y/0}/\partial \lambda_0 < 0$ . For  $j = 3$ ,  $\partial V_{m/0}/\partial c_0 = \partial V_{g/0}/\partial c_0 < 0$  and  $\partial V_{m/0}/\partial \lambda_0 = \partial V_{y/0}/\partial \lambda_0 < 0$ .*

**Proof.** To prove the negative impact of  $c_0$ , note that  $\Phi$  decreases in  $c_0$ . To prove the positive impact of  $\lambda_0$ , note that both  $\alpha$  and  $\Phi$  increase in  $\lambda_0$ . Partial differentiation of the equations (5), (6), (7), (4), (8), (10), and (11) gives the results. ■

This corollary states that the value of the firm decreases in the costs and durations of incubating knowledge. (Note that  $\lambda_0$  is negatively related with the duration of creation.) And the sensitivity of the value to such costs and durations is higher for an established firm than a young firm. The reason for this asymmetric sensitivity is the same as the one for Corollary 1.

Comparative statics with respect to the implementation cost is not as straightforward as the creation cost, and therefore we divide the results in the two corollaries that follow. The next corollary is about comparative statics on the value of established and young firms at their implementation and commercialization stages.

**Corollary 3** *The impact of the implementation cost and duration is as follows:  $\partial V_{m/1}/\partial c_1 < \partial V_{g/1}/\partial c_1 < 0$ ,  $\partial V_{m/2}/\partial c_1 = \partial V_{g/1}/\partial c_1 < 0$ ,  $\partial V_{m/1}/\partial \lambda_1 > \partial V_{g/1}/\partial \lambda_1 > 0$ , and  $\partial V_{m/2}/\partial \lambda_1 = \partial V_{g/2}/\partial \lambda_1 > 0$ .*

**Proof.** To prove the negative impact of  $c_1$ , note that  $\Phi$  decreases in  $c_1$ . To prove the positive impact of  $\lambda_1$ , note that  $\alpha$ ,  $\rho_1$ , and  $\Phi$  are all increasing in  $\lambda_1$ . Together with these facts, partial differentiation of the equations (5), (6), (7), and (10) combined with the equation (3) gives the results. ■

The value of the firm at the implementation and the commercialization stages respond to changes in the implementation cost similarly to changes in the creation cost. This is because once implementation starts, both young and established firms are at an equal footing.

The next corollary is about comparative statics on the value of established and young firms at their creation stage.

**Corollary 4** *The impact of the cost and the duration of implementing on the value of firms at the creation stage is as follows: For  $i = \{g, m\}$ ,*

1.  $\partial V_{i/0}/\partial c_1 < 0$  and  $\partial V_{i/0}/\partial \lambda_1 > 0$ .

2.

$$\partial V_{g/0}/\partial c_1 < \partial V_{m/0}/\partial c_1 < 0$$

if and only if

$$S - \rho_0 \rho_1 \rho_2 - \rho_0 (S - 1) - \rho_0 (r + \lambda_2) \leq 0; \quad (12)$$

3.

$$\partial V_{g/0}/\partial \lambda_1 > \partial V_{m/0}/\partial \lambda_1 > 0$$

if and only if

$$\frac{\alpha \rho_0}{S} (\alpha \rho_2 \Phi + Y) - \frac{c_1}{r} \leq 0. \quad (13)$$

**Proof.** The first part of the corollary can be obtained by differentiating the equations (4), and (8). To prove the second part of the corollary, note that differentiating  $C_1 + \alpha \Phi$  by  $c_1$  gives

$$\frac{\partial (C_1 + \alpha \Phi)}{\partial c_1} = \frac{S - \rho_0 \rho_1 \rho_2 - \rho_0 (S - 1) - \rho_0 (r + \lambda_2)}{(r + \lambda_1) (S - \rho_0 \rho_1 \rho_2 - \rho_0 (S - 1))}.$$

To prove the third part of the corollary, note that differentiating  $C_1 + \alpha \Phi$  by  $\lambda_1$  gives

$$\frac{\partial (C_1 + \alpha \Phi)}{\partial \lambda_1} = \left( \frac{\alpha \rho_0}{S} (\alpha \rho_2 \Phi + Y) - \frac{c_1}{r} \right) \frac{r}{(r + \lambda_1)^2}.$$

Together with equations (10) and (11), the second and the third parts of the corollary follow. ■

A change in the implementation cost  $c_1$  and the duration of implementation stage  $1/\lambda_1$  may or may not impact an established firm more than a young firm. More costly implementation (high  $c_1$  or low  $\lambda_1$ ) impact the value of the firm in the search or creation stage in two ways. First, it increases the cost of implementing the current knowledge in the future. This negative effect is more severe for a young firm than an established firm, because the established firm may not implement the current knowledge at all if it will

turn out not to match its capability. Second, more costly implementation increases the cost of implementing future knowledge. This negative effect is more severe for an established firm because an established firm will stay in the market in the future, whereas a young firm may not. These two effects impact on the difference between the values of an established firm and a young firm in the opposite way. As a consequence, we do not have a clear result.

#### 2.4.2 Value of Progress for Established Firms and Young Firms

We are now going to characterize how the firm value implements across the different firm stages.

**Corollary 5**  $V_{m/1} - V_{m/0} > V_{g/1} - V_{g/0}$ . *The difference in the firm value between implementation stage and creation stage is larger for an established firm than a young firm.*

**Proof.** Subtracting equation (10) from equation (11) gives

$$(V_{m/1} - V_{m/0}) - (V_{g/1} - V_{g/0}) = \frac{S-1}{S} [\rho_1 (Y + \rho_2 \alpha \Phi) - \rho_0 (C_1 + \alpha \Phi)].$$

Noting that  $\alpha \Phi = V_{m/0}$ ,  $Y + \rho_2 \alpha \Phi = V_{m/2}$  and rearranging equation (6) give  $\rho_1 V_{m/2} = V_{m/1} + C_1$ , we get

$$(V_{m/1} - V_{m/0}) - (V_{g/1} - V_{g/0}) = \frac{S-1}{S} [V_{m/1} + (1 - \rho_0) C_1 - \rho_0 V_{m/0}].$$

The right hand side is strictly positive since  $1 - \rho_0 > 0$ ,  $\rho_0 < 1$  and  $V_{m/1} > V_{m/0}$ , and the result follows. ■

Intuitively, if an established firm is implementing knowledge, that implies that the knowledge type is matched with the firm's capability, and the implementation profit is guaranteed. Nevertheless, if a young firm is implementing knowledge, it may not be matched with the firm's capability and therefore it may fail to generate revenues. Therefore, the progress from the creation stage to the implementation stage conveys a stronger signal for an established firm than a young firm.

**Corollary 6**  $V_{m/2} - V_{m/1} < V_{m/2} - V_{g/1}$ . *The difference in the firm value between the harvesting stage and the implementation stage is larger for a young firm than an established firm.*

**Proof.** It immediately follows that  $V_{m/2} = V_{g/2}$  and  $V_{m/1} > V_{g/1}$ . ■

The young firm gains more from becoming an established firm than an established firm does from successfully commercializing a new knowledge. This is because when a young firm progresses to the harvesting stage, this incident signals that the firm has a general ability to stay in the market. One knows that an established firm possesses the ability to stay in the market when it is implementing knowledge. Therefore, the value of a young firm increases more than the value of an established firm, when the firm progresses from the implementation stage to the harvesting stage. This result is based on the same logic in Alti (2003). He argues that the information content of cash flow is bigger and so is the investment sensitivity to cash flow for a young firm than an established firm.

## 2.5 Related Evidence

Proposition 1 and Proposition 2 together imply that the established firm may abandon its knowledge right after the creation, whereas the young firm does not. Glued and Scharfstein (2004) find the evidence consistent with our results. They compare the clinical trial strategies and performance of large, established biopharmaceutical firms to those of early stage firms that have not yet successfully implemented a drug. They find that early stage firms are more likely than established firms to advance drug candidates from Phase I to Phase II clinical trials. However, early stage firms have much less promising clinical results in their Phase II trials and their Phase II drug candidates are also less likely to advance to Phase III and to receive Food and Drug Administration approval.

Kaplan, Sensoys, and Stromberg (2005) find that venture capital funded firms rarely change their business plans. If we interpret the search and creation stages as research and the implementation stage as literally implementation, our model predicts that an established firm spends relatively

more on research and less on implementation than a young firm does. This is not supported by data. For instance NSF (2002) reports that the ratio of implementation to research is 3.1 for firms with not less than 5,000 of employment and this ratio drops to 2.5 for firms with less than 5,000 of employment.<sup>9</sup>

Cockburn and Henderson (2001) examine the performance of drug implementation for clinical research knowledge of 10 multinational pharmaceutical firms. They find that among 585 knowledge with their outcomes known 269 knowledge were abandoned even before any regulatory filings were commenced (Table 2). Unfortunately, the authors do not have data for young pharmaceutical firms, so we cannot compare whether young firms would be slower on relinquishing knowledge that our model predicts. Finally, Danzon, Nicholson and Perreira (2005), using the data on 900 pharmaceutical firms, find that experience do not change the probability that the firm completes the phase 1 clinical trial. Jovanovic (1982) suggests that this probability should be higher for a more experienced firm, because such a firm, on average, should have a better ability than a young firm. Our model may explain this contradiction between the theory and the data, as we demonstrate the opposite force such that a young firm may be more persistent than an established firm. This is because a young firm cannot screen out the knowledge early based on the mismatch between the knowledge type and its own type.

Corollary 6 suggests that the value of young firms is more sensitive to cash flow announcements than that of established firms. Supporting this argument, Zhang (2006) finds that stock return of younger firms are more sensitive to revisions of analysts' earning forecasts (Table III).

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<sup>9</sup>Similarly to Jovanovic (1982), our model predicts that a young firm is less likely to progress to a profitable stage of the knowledge than an established firm. Consistent with this prediction, Macher and Boerner, (2005) based on the data on drug implementation knowledge from CROs (Contract Research Organizations), find that more experienced CROs are less likely to fail in clinical trials than less experienced CROs. We can interpret experience as firm's maturity.

### 3 The Market for Knowledge

So far we have prohibited a firm to sell its knowledge to another firm. In this section, we introduce such an opportunity and study the market for knowledge.

#### 3.1 Stationary Equilibrium

The basic structure of our model with technology transfer is as follows. The entire economy is described by the distribution of firms at different life stages. Let  $G_0(t)$  be the mass of young firms attempting to discover knowledge at time  $t$ ,  $G_1(t, s)$  be the mass of young firms implementing the knowledge  $s$  at time  $t$ , and  $M_i(t, s)$ ,  $i \in \{0, 1, 2\}$  be the mass of established firms with capability  $s$  in stage  $i$  at time  $t$ . Let  $N(t)$  be the mass of new firms entering in the market at time  $t$ . Let  $p(t, s)$  be the price of knowledge with type  $s$  at time  $t$ .

We focus on stationary equilibrium such that the distribution of firm types and the price of each knowledge stays the same over time. Therefore, we drop  $t$  from notation. More formally, we define a stationary equilibrium as follows:

**Definition 5** *A stationary equilibrium with knowledge transfers is a collection of firm masses in the economy  $N, G_0, G_1, M_0(s), M_1(s), M_2(s)$ , knowledge transfer price  $p(s)$  and a collection of firm implementation policies such that*

- *the mass of firms at any given life stage and the price of each knowledge are time invariant, and*
- *given the knowledge price, all firms follow the optimal implementation policies.*

Before starting to characterize the stationary equilibria, it is useful to

introduce the following notation. Let

$$G_1 = \sum_{s=1}^S G_1(s)$$

$$M_1 = \sum_{s=1}^S M_1(s)$$

By the law of large numbers,  $G_1(s) = \frac{1}{S}G_1$ , and  $M_1(s) = \frac{1}{S}M_1$ , for all  $s$ . By symmetry,  $p(s) = p$  for all  $s$ .

### 3.2 Selling Knowledge

We make a few assumptions concerning knowledge transfers. First, we assume that the seller of the knowledge incurs  $\psi \geq 0$  of deadweight cost at the moment of the sales. Second, without loss of generality, we assume that a firm has to make the decision to sell its knowledge at the moment it has created the knowledge. Once it starts to implement the knowledge, it can no longer sell the knowledge to another firm. Third, without loss of generality, we assume that an established firm has to make the decision to buy an created knowledge at the moment the firm's harvesting period has ended.

### 3.3 Life-cycle of Firms

In a presence of the market for knowledge, a young firm starts its life with a decision between purchasing knowledge and creating new knowledge of its own. If the young firm decides to purchase knowledge, the young firm's value is equal to  $V_{g/1} - p$ , and  $V_{g/0}$ , otherwise. Its value is  $V_{g/0}$  while the young firm is engaged in creating new knowledge. Once the young firm creates knowledge, it chooses either implementing the knowledge by itself or selling it to another firm. If the young firm decides to sell its knowledge, the firm gets  $\max\{V_{g/0}, V_{g/1} - p\} + p - \psi$ . If it decides to implement the project, then its value becomes  $V_{g/1}$ . Thus,

$$V_{g/0} = \rho_0 \max \{ \max \{ V_{g/0}, V_{g/1} - p \} + p - \psi, V_{g/1} \} - C_0. \quad (14)$$

If the young firm attempts to implement the project, the project matches with the firm's capability with probability  $1/S$ . In this case, the firm moves

to the harvesting stage and gets  $V_2$ . With the probability  $(S - 1)/S$ , the project does not match with the firm's capability. In this case, at the end of the implementation stage, the firm exits and gets zero. Therefore,

$$V_{g/1} = \frac{\rho_1 V_2}{S} - C_1 \quad (15)$$

Once the harvesting period ends, the established firm chooses either creating new knowledge and getting  $V_{m/0}$  or purchasing the knowledge that fits its own capability by paying  $p$ . If the latter is chosen, the firm proceeds to implement the purchased knowledge and its value becomes  $V_{m/1}$ . Thus,

$$V_2 = Y + \rho_2 \max \{V_{m/0}, V_{m/1} - p\}. \quad (16)$$

If the established firm creates knowledge that fits the firm's capability, the firm proceeds to implement the knowledge and the firm's value becomes  $V_{m/1}$ . Otherwise, the firm sells the knowledge it created and chooses between creating new knowledge or purchasing knowledge created by another firm. Therefore,

$$\begin{aligned} & V_{m/0} \\ = & \frac{1}{S} \rho_0 V_{m/1} + \frac{S-1}{S} \rho_0 (\max \{V_{m/0}, V_{m/1} - p\} + p - \psi) - C_0. \end{aligned} \quad (17)$$

Once the established firm starts implementing the knowledge that matches with the firm's capability, the firm just waits for the harvesting period while incurring the implementation cost. Thus,

$$V_{m/1} = \rho_1 V_2 - C_1. \quad (18)$$

It is worthwhile to mention that the life-cycle described above is a reduced form in the following sense. An established firm naturally has an option to sell its knowledge irrespective of the project type. But we ignore the possibility that the firm may sell the knowledge that matches with the firm's capability. This is because in equilibrium the firm that benefits most from possessing a certain type of knowledge is the one with the matching capability. Besides, selling a project costs  $\psi$ . Therefore, the equilibrium price of the project should not be so high that even the firm with the matching capability is willing to sell it to another firm.

### 3.4 Non-Existence of Equilibrium in Which Young Firms Buy Knowledge

We are now going to show that a young firm does not buy any knowledge in equilibrium. First, we will show that a young firm should not buy knowledge from another young firm and second, we will show that a young firm should not buy knowledge from an established firm.

If a young firm were to buy knowledge from another young firm to creating one in equilibrium, the buying young firm needs to be willing to pay  $p$  to another young firm instead of creating new knowledge, as well as another young firm is willing to create knowledge and then sell it later. The first condition is summarized to

$$V_{g/1} - p \geq V_{g/0}. \quad (19)$$

That is, a young firm prefers buying created knowledge instead of creating its own. The second condition requires the following two conditions:

$$V_{g/1} - p \leq V_{g/0} \quad (20)$$

and

$$\max \{V_{g/0}, V_{g/1} - p\} + p - \psi \geq V_{g/1}. \quad (21)$$

**Proposition 4** *There does not exist equilibrium in which a young firm buys a project from another young firm.*

**Proof.** Combining equations (19) and (20) gives  $V_{g/1} - p = V_{g/0}$ . As a result,  $\max \{V_{g/0}, V_{g/1} - p\} = V_{g/0}$ . Nevertheless, this equation together with equation (21) implies  $-\psi \geq 0$ , which is contradiction. Q.E.D.

Intuition behind this proposition is straightforward. If a young firm buys knowledge from another young firm, such a trade does not change the probability that the knowledge will be successfully implemented. Nevertheless, such a trade wastes transaction cost  $\psi$ . Therefore, it is inefficient and a gain from trade does not exist.

For a young firm's purchase of knowledge from an established firm to occur, two conditions are needed. First, a young firm prefers buying created

knowledge to incubating its own knowledge. That is, equation (19) needs to hold. Second, an established firm prefers incubating own knowledge to purchasing knowledge which matches the firm's capability. Thus,

$$V_{m/0} \geq V_{m/1} - p. \quad (22)$$

The next proposition states that these two conditions are not compatible.

**Proposition 5** *There does not exist equilibrium in which a young firm buys knowledge from an established firm.*

**Proof.** See Appendix.

Intuition behind this proposition is as follows. The benefits of buying knowledge instead of creating new knowledge are saving the creation cost and moving to the implementation stage without delays. This type of gains exists no matter which a young or established firm is purchasing knowledge. In addition, if the firm that buys new knowledge is an established firm, it can selectively acquire the created knowledge that matches its own capability and can therefore avoid the risk of creating knowledge that does not match the firm's capability. As a consequence of this additional gain, buying knowledge always brings a bigger benefit to established firms than young firms. Thus, if a young firm would be buying knowledge, so should be an established firm and no established firm should create knowledge of its own. As a consequence, a young firm should not buy knowledge from an established firm in equilibrium.

### 3.5 Exploration versus Exploitation

Given that only established firms buy knowledge in equilibrium, it is useful to distinguish two regimes. In the first regime, established firms purchase knowledge from young firms. Unsold knowledge are implemented by young firms. We call this regime *exploitation* because the established firms focus on exploiting such opportunities and are not engaged in creating knowledge. In the second regime, established firms create knowledge. We call this regime *exploration* because the established firms also explore opportunities to discover new knowledge.

### 3.6 Characteristics of Stationary Equilibrium

It is convenient to point out a necessary condition for an equilibrium to have a positive mass of established firms. Without young firm's growth, the mass of established firms declines over time because of a positive death rate  $\mu$ . Therefore,

$$p - \psi + V_{g/0} \leq V_{g/1}. \quad (23)$$

The stationary equilibrium is characterized by the following system of equations.

$$\mu G_0 + \lambda_0 (1 - H_g) G_0 = N, \quad (24)$$

$$(\mu + \lambda_1) G_1 = \lambda_0 G_0 (1 - H_g), \quad (25)$$

$$(\mu + \lambda_1) M_1 = \lambda_0 \left( \frac{S-1}{S} M_0 H_m^0 + \frac{1}{S} M_0 \right) + \lambda_2 M_2 H_m^2, \quad (26)$$

$$(\mu + \lambda_2) M_2 = \frac{\lambda_1 G_1}{S} + \lambda_1 M_1, \quad (27)$$

and

$$\left( \mu + \frac{\lambda_0}{S} + \frac{\lambda_0 (S-1)}{S} H_m^0 \right) M_0 = \lambda_2 M_2 (1 - H_m^2) + \frac{\lambda_0 (S-1)}{S} (1 - H_m^0) M_0, \quad (28)$$

where  $H_g$  is the proportion of knowledge created by young firms and sold to established firms,  $H_m^2$  is the ratio of established firms that face the end of the harvesting period and buy knowledge right away, and  $H_m^0$  is the ratio of established firms that have just created a non-matched knowledge and buy knowledge from another firm. Equation (24) ensures the outflow and the inflow of incubating new firms are the same. Note that among the young firms that have just created knowledge, the fraction of  $H_g$  is sold to established firms and the rest will implement their knowledge by their own. Then, equation (25) ensures that the outflow and the inflow of implementing young firms are the same. Note that the right hand side of equation (26) is the inflow of implementing established firms. It consists of the mass of knowledge transferred from young firms, the mass of knowledge the same established firms have created and are going to implement, and the mass of knowledge transferred from established firms to other established firms.

Then, equation (26) ensures that the outflow and the inflow of implementing established firms are the same. Equation (27) ensures that the outflow and the inflow of harvesting firms are the same. The left hand side of equation (28) is the outflow of incubating established firms and it consists of those who have just died, those who have just created knowledge and are going to implement them by their own, and those who have just created knowledge and sold their knowledge to other established firms. The right hand side of equation (28) is the inflow of established firms that attempt to discover knowledge and it is equal to the mass of established firms that finished harvesting minus those who have just bought knowledge from either young or other established firms. Finally, we also need the knowledge market clearing condition for knowledge as follows:

$$\frac{\lambda_0 (S - 1)}{S} M_0 H_m^0 + \lambda_2 M_2 H_m^2 = \lambda_0 H_g G_0 + \frac{\lambda_0 (S - 1)}{S} M_0.$$

Note that the left hand side is established firms' demand for knowledge and the right hand side is supply of knowledge from young and established firms.

### 3.6.1 Exploitation Equilibrium (Established firms specialize in implementation)

We begin with examining if there exists an equilibrium in which established firms specialize in implementing knowledge and create no knowledge. In such an equilibrium,

$$V_{m/1} - p \geq V_{m/0},$$

and  $M_0 = 0$ . We also need young firms willing to sell their knowledge and therefore

$$V_{g/0} + p - \psi \geq V_{g/1}.$$

Together with equation (23),

$$V_{g/0} + p - \psi = V_{g/1}.$$

We call such an equilibrium *exploitation equilibrium*. The following proposition ensures the existence of exploitation equilibrium.

**Proposition 6** *If*

$$\psi \leq \frac{(S-1)(1-\rho_0)\rho_1(Y-\rho_2C_0-\rho_0\rho_2C_1)}{S(1-\rho_0)+\rho_0(1-\rho_1\rho_2)-\frac{S-1}{S}(1-\rho_0)\rho_0\rho_1\rho_2} \quad (29)$$

*then there exists exploitation equilibrium such that  $M_1, M_2 > 0$ . In this equilibrium, the knowledge price is given by*

$$p = \frac{\rho_1(1-\rho_0)Y + S(1-\rho_1\rho_2)(\psi + C_0) - (1-\rho_0)[S(1-\rho_1\rho_2) + \rho_1\rho_2]C_1}{S(1-\rho_1\rho_2) + \rho_1\rho_2(1-\rho_0)} \quad (30)$$

**Proof.** See Appendix.

For the exploitation equilibrium to exist, the transfer cost  $\psi$  needs to be sufficiently low, so that a young firm does not require too high a price to sell its knowledge to an established firm. Not surprisingly, the knowledge price is increasing in the cost of creating knowledge. The knowledge price is decreasing in the implementation cost, as a young firm is willing to sell its knowledge at a lower price if not selling and implementing the knowledge results in a higher loss. The knowledge price is increasing in the revenues, as a young firm is less willing to sell its knowledge if not selling can potentially bring in higher revenues and therefore the price needs to increase in order to induce the young firm to sell its knowledge to an established firm. The knowledge price is decreasing in  $S$  because an increase in  $S$  makes it more likely that the young firm's attempt to implement knowledge may fail.<sup>10</sup> As a result, a young firm is more willing to sell its knowledge at a lower price when  $S$  is bigger.

Given the death rate  $\mu$ , the entry mass  $N$ , and the rates to move to the next stage,  $\lambda_0$ ,  $\lambda_1$  and  $\lambda_2$ , one can solve for the complete distribution  $G_0$ ,  $G_1$ ,  $M_2$  and  $M_1$ . In what follows, we characterize the stationary equilibrium.

<sup>10</sup>Differentiating  $p$  with respect to  $S$  gives

$$\frac{dp}{dS} = \frac{\rho_1(1-\rho_1\rho_2)(\rho_2\psi - Y)}{(S(1-\rho_1\rho_2) + \rho_1\rho_2)^2}.$$

This is negative because  $Y > \rho_2\psi$  by the existence condition for the exploitation equilibrium described in Proposition 6.

**Proposition 7** *In the exploitation equilibrium, the rate of knowledge transfers defined by  $\lambda_2 M_2 / \lambda_0 G_0$  is decreasing in  $\mu$  and  $S$  and increasing in  $\lambda_1$  and  $\lambda_2$ .*

**Proof.** See Appendix.

Intuitions behind this proposition are as follows. When  $\mu$  and  $S$  are high, less firms become established and, as a result, established firms' demand for knowledge is low and less knowledge transfers occur. The positive relation between  $S$  and the acquisition rate is paradoxical. If  $S$  is high, commercialization attempts by new firms are more likely to fail. Nevertheless, the rate of such an attempt is higher when  $S$  is high. When  $\lambda_1$  and  $\lambda_2$  are high, established firms come back to the knowledge market quickly and, as a result, demand for knowledge increase. As a consequence, the knowledge transfer rate increases.

**Proposition 8** *In the exploitation equilibrium, the values of established firms  $V_2$  and  $V_{m/1}$  are both increasing in  $S$ . The values of young firms  $V_{g/0}$  and  $V_{g/1}$  are both decreasing in  $S$ .*

**Proof.** See Appendix.

Intuitions behind this proposition are similar to how entry deterrence asymmetrically influences incumbents and new entrants. When  $S$  is high, it is unlikely that a young firm can successfully implement the knowledge it has just created. Implementing by itself is an outside option of the young firm when it negotiates the price of selling the knowledge to an established firm. If this outside option is not attractive, the bargaining power of the young firm falls and so does the price of the knowledge. Opposite to young firms, an established firm can buy knowledge at a cheaper price if  $S$  is high, and therefore the value of the established firm rises.

### 3.6.2 Exploration Equilibrium (An established firm buys knowledge from another established firm.)

We are now going to examine if there exists a stationary equilibrium in which some established firms create knowledge and trade their knowledge among

established firms. Note that there always exists a stationary equilibrium in which established firms create and they do not sell their knowledge at all, if the transfer cost  $\psi$  is sufficiently high. We have analyzed this case in Section 2.

We begin our analysis by describing the necessary conditions for a stationary equilibrium in which some established firms create knowledge. The first condition is that an established firm needs to be willing to create knowledge instead of buying an knowledge, that is,  $V_{m/0} \geq V_{m/1} - p$ . The second condition is that an established firm is also willing to buy knowledge, that is,  $V_{m/1} - p \geq V_{m/0}$ . Therefore,

$$p = V_{m/1} - V_{m/0}. \quad (31)$$

In addition, an established firm should be willing to sell its knowledge instead of discarding the knowledge, that is,  $V_{m/0} \leq p - \psi + V_{m/0}$ . Combining with equation (23) gives

$$0 \leq p - \psi \leq V_{g/1} - V_{g/0}. \quad (32)$$

The following proposition characterizes the stationary equilibrium with established firms incubating knowledge.

**Proposition 9** *If*

$$\begin{aligned} & [S(1 - \rho_0) + \rho_0(1 - \rho_1\rho_2)]\psi \\ & \leq S(1 - \rho_0)(\rho_1Y - C_1) + S(1 - \rho_1\rho_2)C_0, \end{aligned} \quad (33)$$

and

$$\frac{(S - 1)(1 - \rho_0)\rho_1(Y - \rho_2C_0 - \rho_0\rho_2C_1)}{S(1 - \rho_0) + \rho_0(1 - \rho_1\rho_2) - \frac{S-1}{S}(1 - \rho_0)\rho_0\rho_1\rho_2} \leq \psi, \quad (34)$$

then there exists a stationary equilibrium in which established firms create knowledge. The equilibrium price of the knowledge is

$$p = \frac{S(1 - \rho_0)(\rho_1Y - C_1) + S(1 - \rho_1\rho_2)C_0 + (S - 1)\rho_0(1 - \rho_1\rho_2)\psi}{S(1 - \rho_0\rho_1\rho_2)}. \quad (35)$$

**Proof.** See Appendix.

The first condition (33) ensures that  $\psi$  is low enough for established firms to be willing to sell their unmatched knowledge. The second condition (34) ensures that  $\psi$  is high enough for young firms to not always sell their knowledge to established firms. Similarly to the results in the exploitation equilibrium, the knowledge price is increasing in the revenue, the cost of creating knowledge and the transaction cost and decreasing in the implementation cost.

The following corollary summarizes the necessary condition for existence of the two different types of stationary equilibrium we have discussed.

**Corollary 7** *Suppose that equation (33) holds. Then, if equation (34) holds, then an exploration equilibrium exists but an exploitation equilibrium does not. Otherwise, an exploitation equilibrium exists but an exploration equilibrium does not.*

**Proof.** It suffices to note that condition (29) and condition (33) are mutually exclusive. Q.E.D.

This corollary implies that there does not exist multiple equilibria given exogenous variables fixed. In particular, if the transaction cost is low, established firms buy knowledge from young firms. If the transaction cost is moderate, then established firms buy knowledge from other established firms. If transaction cost of acquisition is high, the market for knowledge does not exist in equilibrium.

We have established the property of equilibrium knowledge price given an equilibrium regime. The following corollary states that even the equilibrium regime switches due to changes in exogenous variables, the property of equilibrium knowledge price still remains.

**Corollary 8** *Knowledge price  $p$  is increasing in  $Y$ ,  $\psi$  and  $C_0$  and decreasing in  $C_1$ .*

**Proof.** Let  $p_i$  be the knowledge price under exploitation equilibrium defined by equation (30) and  $p_r$  be the knowledge price under exploration equilib-

rium defined by equation (31). To prove this corollary, note that  $p_i = p_r$  when inequalities (29) and (33) are replaced by equality. Q.E.D.

### 3.7 Related Evidence

Consistent with corollary 7, Lamoreaux and Sokoloff (2003) document that in 19th century U.S., the emergence of specialized inventors followed the introduction of patent system. The patent system lowered the transaction cost of selling knowledge by not only establishing property rights to knowledge but also creating intermediaries for the market of knowledge, such as patent lawyers.

In exploitation equilibrium, our model implies that sold knowledge had a greater life expectancy and also a higher profitability than non-transferred knowledge. This is because sold knowledge are implemented by the firms with the matching capabilities, whereas non-transferred knowledge may not be. Consistent with this prediction, Ravenscraft and Scherer (1987) find that acquired lines of business are of higher quality (usually measured by profitability) than those not acquired. Churchill (1955) finds that transferred businesses are more profitable than non-transferred businesses. Another interpretation of our model is to identify a venture capital funding as an acquisition. Gorman and Sahlman (1989) and Hellmann and Puri (2002) document that venture capitalists frequently replace the management of their portfolio firms suggesting that the firms' business were pushed forward by somebody different from the original incubator of the business. Under this interpretation, our model predict that venture capital backed firms perform better than other firms. Consistent with this prediction, Jain and Kini (2002) find that a VC-backed IPO firm will survive longer than non-VC-backed IPO firms. Hellmann and Puri (2000) find that VC-backed firms grow faster than non-VC backed firms.

Finally, our model also predicts that a young firm is more likely to fail than an established firm, as the former firm has to implement knowledge without knowing if the knowledge matches the firm's capability. Jain and Kini (1994) find that IPO firms operating performance is worse than match-

ing firm performance. Given that many IPO firms are new firms, their evidence suggest that implementation by new firms perform worse than that by established firms as consistent with our model.

## 4 Sequential Learning

So far we have assumed that the young firm will exit the market right after it fails in implementing knowledge. We now allow the young firm to stay in the market after previous failures at implementing knowledge. In this way the firm can use the knowledge it gained about its capability by becoming more selective about the types of knowledge it chooses to implement. In particular, the firm will avoid implementing knowledge of a certain type if it failed at implementing this particular type of knowledge in the past. This way the young firm avoids wasting the implementation cost for knowledge type that does not match its capability. On the other hand, we may think of the model with firm exit after the first failure at implementing as representing some financial friction that prevents funding for future knowledge.

Let  $V_{g/0}[N]$  be the value of a young firm at the creation stage when it has failed to implement exactly  $N$  different types of knowledge in the past for  $N = 0, 1, \dots, S - 2$ . Then the firm will discover knowledge of a type that it failed implementing in the past with probability  $N/S$  and it will discover knowledge of a new type with probability  $(S - N)/S$ . Similarly let  $V_{g/1}[N]$  be the value of a young firm at the implementation stage. Then for  $N = 0, 1, \dots, S - 2$  we get

$$\begin{aligned}
 V_{g/0}[N] &= \rho_0 \frac{N}{S} \max \{p - \psi + V_{g/0}[N], V_{g/1}[N] - \psi\} \\
 &\quad + \rho_0 \frac{S - N}{S} \max \{p - \psi + V_{g/0}[N], V_{g/1}[N]\} - C_0 \quad (36)
 \end{aligned}$$

The first term represents the impact of the firm's knowledge about past implementation failures. The firm will choose to either sell the knowledge and discover a new knowledge, getting  $p - \psi + V_{g/0}[N]$  or sell the knowledge and buy knowledge from another firm, getting  $p - \psi + V_{g/1}[N] - p = V_{g/1}[N] - \psi$ . The second term represents the firm's policy towards knowledge type for

which it has no prior experience. The firm will choose to either sell the knowledge and discover a new knowledge, getting  $p - \psi + V_{g/0} [N]$  or implement the new knowledge, getting  $V_{g/1} [N]$ . The third term represents the cost of incubating knowledge, similarly with the model in previous sections. Note that

$$V_{g/0} [S - 1] = V_{m/0}$$

since once the firm failed to implement  $S - 1$  different types of knowledge, the one type of knowledge that it never failed to implement before should match the firm's capability, so the firm will infer its capability. We now derive the firm value at the implementation stage. In particular, if the young firm has failed in implementing  $N$  different types of knowledge, then the firm will only implement among the other  $S - N$  types of knowledge. Then, conditional on this pre-screening, for  $N = 0, 1, \dots, S - 2$  we get

$$V_{g/1} [N] = \rho_1 \frac{1}{S - N} V_2 + \rho_1 \frac{S - N - 1}{S - N} \max \{V_{g/0} [N + 1], V_{g/1} [N + 1] - p, 0\} - C_1 \quad (37)$$

and

$$V_{g/1} [S - 1] = V_{m/1}$$

The first term represents the value obtained if the knowledge type matches the firm's capability. The second term represents the value obtained otherwise. Even if the implementation fails, the firm learns that the type of the knowledge it just failed to implement does not match its capability, so it has now failed at implementing  $N + 1$  different types of knowledge. The firm will then choose to discover a new knowledge to get  $V_{g/0} [N + 1]$  or acquiring knowledge to get  $V_{g/1} [N + 1] - p$ , or exiting the market to get 0.

**Proposition 10** *Suppose that in equilibrium*

$$\begin{aligned} 0 &\leq p - \psi, \\ p - \psi + V_{g/0} [0] &\leq V_{g/1} [0] \end{aligned}$$

and

$$V_{m/0} \leq V_{m/1} - p.$$

Then,

$$V_{g/i} [N - 1] \leq V_{g/i} [N], \text{ for } N = 1, \dots, S - 1 \text{ and } i = 0, 1.$$

**Proof.** See Appendix. ■

The intuition behind the proposition is straightforward. Failing in past attempts to implement knowledge allows the firm to narrow down the set of possible capabilities it has. Therefore, the firm's value will increase with the number of past failures.

**Proposition 11** For  $N = 0, \dots, S - 2$ ,

$$V_{g/1} [N] - V_{g/0} [N] < V_{g/1} [N + 1] - V_{g/0} [N + 1].$$

**Proof.** See Appendix. ■

This proposition confirms the robustness of our proposition 4 and proposition 5. According to this proposition, the value of implementing the knowledge that may match the firm's capability increases faster than the value of creating new knowledge. As a result, as firms experiences more failures in implementing knowledge, they become more willing to pay and buy knowledge instead of creating its own. Therefore, in equilibrium, older firms are more likely to buy knowledge from younger firms.

## 5 Conclusion

This paper models the difference of an established firm and a young firm by the knowledge of own capability; an established firm knows its own capability but a young firm does not know. This simple difference is enough to generate rich predictions. Our model implies that an established firm tends to abandon knowledge earlier than a young firm. This difference arises because an established firm discovers that knowledge may not fit its capability at the early stage but a young firm cannot apply such an abandonment criterion not knowing its own capability. Our model also implies that a new firm may sell its knowledge to an established firm, but not the other way around, and sold knowledge should implemented better than unsold knowledge.

We also find that the transaction cost of selling knowledge determines has an important impact on the market for knowledge. When the transaction cost is moderate, established firms trade their knowledge each other. When the transaction cost is low, established firms do not create knowledge and buy it from young firms. Finally, our model suggests that it is important to distinguish the question, who creates knowledge, from the question, who implements knowledge. Our model suggest that who implements knowledge is less likely to be who creates knowledge recently, due to reduction of the transaction cost.

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## Appendix

This appendix gathers the proofs of the propositions for that we did not provide the proofs in the text.

### Proof of Proposition 1

It suffices to show that

$$V_{m/1}(d^*) \geq V_{m/1}(d), \forall d \in 2^{T^S}.$$

Suppose that the type of the created knowledge is  $s_p = s_f$ . Let  $\hat{d}, d$  be such that  $\hat{d}(s_p) = 1, d(s_p) = 0$ . We will show that  $V_{m/1}(\hat{d}) \geq V_{m/1}(d)$ . Note that we do not constraint the firm policy in subsequent decisions. If the firm does not implement this knowledge, the value of the firm is equal to  $V_{m/0}(d)$ . If the firm does implement this knowledge, the value of the firm is equal to

$$\begin{aligned} V_{m/1}(\hat{d}) &= \rho_1 Y - C_1 + \rho_2 \rho_1 V_{m/0}(\hat{d}) \implies \\ V_{m/1}(\hat{d}) &\geq \rho_1 Y - C_1 + \rho_2 \rho_1 V_{m/0}(d) \end{aligned}$$

The second line follows from the fact that the continuation decisions of the firm are unconstrained, so given  $d$  the firm can do at least as well in policy  $\hat{d}$  by replicating  $d$  in all subsequent histories. We want to show

$$\rho_1 Y - C_1 > (1 - \rho_2 \rho_1) V_{m/0}(d)$$

Let

$$\int_0^{\lambda_1^{-1} + \lambda_2^{-1}} z e^{-rt} dt = \rho_1 Y - C_1.$$

That is,  $z$  is per period cash flow for the expected duration of implementation plus commercialization periods that is equivalent to the payoff of implementing the matched knowledge. Solving this equation gives

$$z = \frac{r(\rho_1 Y - C_1)}{1 - e^{-r(\lambda_1^{-1} + \lambda_2^{-1})}}.$$

Since [the perpetuity of the cream skimming period should be larger than the value of the firm of non-skimming period,]

$$z > r V_{m/0}(d). \tag{38}$$

Integrating both sides of the equation (38) from 0 to  $\lambda_0^{-1} + \lambda_0^{-2}$  give

$$\rho_1 Y - C_1 > \left(1 - e^{-r(\lambda_1^{-1} + \lambda_2^{-1})}\right) V_{m/0}(d).$$

Now we show that

$$e^{-r(\lambda_1^{-1} + \lambda_2^{-1})} < \rho_1 \rho_2$$

which is alternatively

$$e^{r/\lambda_2} e^{r/\lambda_3} > \rho_2^{-1} \rho_3^{-1},$$

which is true, since  $e^x > 1 + x$ , for  $x > 0$ .

Suppose now that the type of the created knowledge is  $s_p \neq s_f$ . Let  $\hat{d}, d$  be such that  $\hat{d}(s_p) = 0, d(s_p) = 1$ . We will show that  $V_{m/1}(\hat{d}) \geq V_{m/1}(d)$ . If the firm does not implement this knowledge, the value of the firm is equal to  $V_{m/0}(\hat{d})$ . If the firm does implement this knowledge, the firm's value is less than  $V_{m/0}(\hat{d})$  because it is equal to the present value of making loss plus the discounted value of  $V_{m/0}(d)$ , which is not more than  $V_{m/0}(\hat{d})$ . Since  $\hat{d} = d^*$ , the proof is complete. Q.E.D.

## Proof of Proposition 2

Suppose that the young firm has just created knowledge. The value from implementing the knowledge given the policy  $d$  is

$$V_{g/1}(d) = -C_1 + \rho_1 \left( \frac{1}{S} V_{g/2}(d) + \frac{S-1}{S} 0 \right) = -C_1 + \frac{1}{S} \rho_1 V_{m/2},$$

where the second equality follows by noting that a young firm becomes established when it starts generating revenue, since it infers its type from the knowledge type that it created. This expression is positive by Assumption 1, so it remains to prove that the value from implementing immediately is greater than the value from not implementing the knowledge  $V_{g/0}(d)$ .

$$\begin{aligned} V_{g/0} &= -C_0 + \rho_0(-C_1 + \rho_1 V_{g/2}) \implies \\ V_{g/2} - V_{g/0} &= V_{g/2}(1 - \rho_0) + C_0 \end{aligned}$$

The last expression is positive, so the proof is complete. Q.E.D.

### Proof of Proposition 3

According to the asset pricing formula, the value of the firm depending on its knowledge cycle is

$$(r + \lambda_0) V_{i/0} = -c_0 + \frac{\lambda_0}{S} V_{i/1} + \frac{\lambda_0 (S-1)}{S} V_{i/0} \quad i \in \{g, m\}, \quad (39)$$

$$(r + \lambda_1) V_{g/1} = -c_1 + \frac{\lambda_1 V_{g/2}}{S}, \quad (40)$$

$$(r + \lambda_1) V_{m/1} = -c_1 + \lambda_1 V_{m/2}, \quad (41)$$

and

$$(r + \lambda_2) V_{i/2} = y + \lambda_2 V_{m/0} \quad i \in \{y, m\}.$$

As there are eight independent equations and eight unknowns, we can obtain the explicit formula of each firm value. Solving this system yields the results. Q.E.D.

### Proof of Proposition 5

We prove this proposition by contradiction. Suppose that equations (19) and (22) hold simultaneously. Then,

$$V_{g/1} - V_{g/0} \geq V_{m/1} - V_{m/0}. \quad (42)$$

If  $V_{m/0} + p - \psi \geq \max \{V_{m/0}, V_{m/1} - p\}$ , that is, an established firm is willing to be acquired, then substituting the definition of  $V_{g/0}$  and  $V_{m/0}$  into equation (42) and rearranging the terms given equations (19) and (22), we have

$$\begin{aligned} & V_{g/1} - \rho_0 \max \{V_{g/1} - \psi, V_{g/1}\} \\ & \geq V_{m/1} - \frac{1}{S} \rho_0 V_{m/1} - \frac{S-1}{S} \rho_0 (V_{m/0} + p - \psi). \end{aligned}$$

As  $\psi > 0$ , after rearranging we can rewrite this inequality as

$$(1 - \rho_0) (V_{g/1} - V_{m/1}) \geq \frac{S-1}{S} \rho_0 (V_{m/1} - V_{m/0} - p + \psi) \geq 0.$$

The second inequality follows because  $V_{m/1} \geq V_{m/0} + p - \psi$  (i.e. implementing the knowledge that matches the firm's capability is more profitable than selling it to a third party). Nevertheless, by equations (15) and (18),  $V_{m/1} > V_{g/1}$  since  $S > 1$  and  $V_2 > 0$ . This is contradiction. Q.E.D.

## Proof of Proposition 6

As previously argued, equation (23) needs to hold in a stationary equilibrium such that  $M_1, M_2 > 0$ . It also must hold that  $p + V_{g/0} \geq \psi + V_{g/1}$  so that a young firm is willing to sell its created knowledge in a stationary equilibrium. As a consequence,

$$p = \psi + V_{g/1} - V_{g/0}. \quad (43)$$

In a stationary equilibrium of interest with  $M_0 = 0$ , the system of the following Bellman equations holds:

$$\begin{aligned} V_2 &= Y + \rho_2(V_{m/1} - p), \\ V_{m/1} &= -C_1 + \rho_1 V_2, \\ V_{g/1} &= -C_1 + \frac{\rho_1}{S} V_2, \text{ and} \\ V_{g/0} &= -C_0 + \rho_0 V_{g/1}. \end{aligned} \quad (44)$$

Solving for  $V_2$ ,  $V_{m/1}$  and then  $V_{g/1}$  gives

$$V_2 = \frac{Y - \rho_2(p + C_1)}{1 - \rho_1 \rho_2}, \quad (45)$$

$$V_{m/1} = -C_1 + \rho_1 \frac{Y - \rho_2(p + C_1)}{1 - \rho_1 \rho_2} \text{ and} \quad (46)$$

$$V_{g/1} = -C_1 + \frac{\rho_1}{S} \frac{Y - \rho_2(p + C_1)}{1 - \rho_1 \rho_2}. \quad (47)$$

Sticking  $V_{g/1}$  into equation (43) and solving for  $p$  give equation (30).

It remains to show that an established firm is willing to pay this price to start implementing knowledge immediately, instead of incubating knowledge by itself. Recall that when there exists a market for knowledge transfers, the established firm that just lost its harvesting knowledge has value

$$V_{m/0} = -C_0 + \frac{1}{S} \rho_0 V_{m/1} + \frac{S-1}{S} \rho_0 (p - \psi + \max \{V_{m/0}, V_{m/1} - p\})$$

In the equilibrium of interest with  $M_0 = 0$  it must be that  $V_{m/0} \leq V_{m/1} - p$  since the established firm prefers buying an created knowledge, so we get

$$V_{m/0} = -C_0 + \frac{1}{S} \rho_0 V_{m/1} + \frac{S-1}{S} \rho_0 (V_{m/1} - \psi)$$

Substituting the equilibrium price  $p$  and  $V_{m/1}$  we get

$$V_{m/0} = \frac{S\rho_0\rho_1Y - [S - (S-1)\rho_1\rho_2(1-\rho_0)](C_0 + \rho_0C_1) - [S-1 + \rho_1\rho_2 + (1-\rho_0)\frac{S-1}{S}\rho_1\rho_2]\rho_0\psi}{S(1-\rho_1\rho_2) + \rho_1\rho_2(1-\rho_0)}$$

Then given price, the necessary condition for an exploitation equilibrium will be

$$V_{m/0} \leq V_{m/1} - p$$

Substituting the expressions for  $V_{m/0}$ ,  $V_{m/1}$  and  $p$  we get

$$\psi \leq \frac{(S-1)(1-\rho_0)\rho_1(Y - \rho_2C_0 - \rho_0\rho_2C_1)}{S(1-\rho_0) + \rho_0(1-\rho_1\rho_2) - \frac{S-1}{S}(1-\rho_0)\rho_0\rho_1\rho_2}$$

then it is optimal for the established firm to buy created knowledge.Q.E.D.

### Proof of Proposition 7

Sticking  $M_0 = 0$ ,  $H_m^2 = 1$ ,  $H_m^0 = 0$  into the system of equations (24)-(28) we get

$$\begin{aligned} [\mu + \lambda_0(1 - H_g)]G_0 &= N \\ (\mu + \lambda_1)G_1 &= \lambda_0(1 - H_g)G_0 \\ (\mu + \lambda_1)M_1 &= \lambda_2M_2 \\ (\mu + \lambda_2)M_2 &= \frac{\lambda_1G_1}{S} + \lambda_1M_1 \end{aligned}$$

The market clearing condition for the exploitation equilibrium becomes

$$\lambda_2M_2 = \lambda_0H_gG_0$$

Therefore, the ratio of knowledge transfers is

$$H_g = \frac{\lambda_2M_2}{\lambda_0G_0}$$

Solving gives the stationary distribution of firms expressed by equations (48).

$$\begin{aligned}
G_0 &= \frac{S\mu(\mu + \lambda_1 + \lambda_2) - \lambda_1\lambda_2}{(\mu + \lambda_0)[S\mu(\mu + \lambda_1 + \lambda_2) - \lambda_1\lambda_2] - \lambda_0\lambda_1\lambda_2}N, \\
G_1 &= \frac{S\mu\lambda_0(\mu + \lambda_1 + \lambda_2)}{(\mu + \lambda_1)\{(\mu + \lambda_0)[S\mu(\mu + \lambda_1 + \lambda_2) - \lambda_1\lambda_2] - \lambda_0\lambda_1\lambda_2\}}N, \\
M_0 &= 0, \\
M_1 &= \frac{\lambda_0\lambda_1\lambda_2}{(\mu + \lambda_1)\{(\mu + \lambda_0)[S\mu(\mu + \lambda_1 + \lambda_2) - \lambda_1\lambda_2] - \lambda_0\lambda_1\lambda_2\}}N, \\
M_2 &= \frac{\lambda_0\lambda_1}{(\mu + \lambda_0)[S\mu(\mu + \lambda_1 + \lambda_2) - \lambda_1\lambda_2] - \lambda_0\lambda_1\lambda_2}N, \tag{48}
\end{aligned}$$

Therefore, the ratio of knowledge transfers is

$$H_g = \frac{\lambda_2 M_2}{\lambda_0 G_0} = \frac{\lambda_1 \lambda_2}{S\mu(\mu + \lambda_1 + \lambda_2) - \lambda_1 \lambda_2},$$

which is decreasing in  $\mu$  and  $S$ , and increasing in  $\lambda_1$  and  $\lambda_2$ . Q.E.D.

### Proof of Proposition 8

Sticking equation (30) into equation (46) for  $p$  gives the closed form solution of  $V_{m/1}$

$$V_{m/1} = \frac{S[\rho_1 Y - \rho_1 \rho_2(\psi + C_0)] - [S - (S - 1)\rho_1 \rho_2(1 - \rho_0)]C_1}{S(1 - \rho_1 \rho_2) + \rho_1 \rho_2(1 - \rho_0)},$$

Sticking equation (30) into equation (45) for  $p$  gives the closed form solution of  $V_2$

$$V_2 = \frac{S[Y - \rho_2(\psi + C_0) - \rho_0 \rho_2 C_1]}{S(1 - \rho_1 \rho_2) + \rho_1 \rho_2(1 - \rho_0)}.$$

Sticking equation (30) into equation (47) for  $p$  gives the closed form solution of  $V_{g/1}$

$$V_{g/1} = \frac{\rho_1 Y - \rho_1 \rho_2(\psi + C_0) - [S(1 - \rho_1 \rho_2) + \rho_1 \rho_2]C_1}{S(1 - \rho_1 \rho_2) + \rho_1 \rho_2(1 - \rho_0)}.$$

Sticking equation (30) into equation (44) for  $p$  gives the closed form solution of  $V_{g/0}$

$$V_{g/0} = \frac{\rho_0 \rho_1 Y - \rho_0 \rho_1 \rho_2 \psi - [S(1 - \rho_1 \rho_2) + \rho_1 \rho_2]C_0 - [S\rho_0(1 - \rho_1 \rho_2) + \rho_0 \rho_1 \rho_2]C_1}{S(1 - \rho_1 \rho_2) + \rho_1 \rho_2(1 - \rho_0)}.$$

It is obvious that both  $V_{g/0}$  and  $V_{g/1}$  are decreasing in  $S$ . Differentiating  $V_2$  and  $V_{m/1}$  by  $S$  gives

$$\frac{dV_2}{dS} = \frac{[Y - \rho_2(\psi + C_0) - \rho_0\rho_2C_1]\rho_1\rho_2(1 - \rho_0)}{[S(1 - \rho_1\rho_2) + \rho_1\rho_2(1 - \rho_0)]^2} > 0.$$

and

$$\frac{dV_{m/1}}{dS} = \rho_1 \frac{dV_2}{dS} > 0.$$

Therefore, the proposition follows. Q.E.D.

### Proof of Proposition 9

As previously argued, equation (31) needs to hold in a stationary equilibrium such that  $M_1, M_2 > 0$ . In a stationary equilibrium of interest, the system of the following Bellman equations holds:

$$\begin{aligned} V_2 &= Y + \rho_2 V_{m/0}, \\ V_{m/0} &= -C_0 + \rho_0 \left( \frac{V_{m/1}}{S} + \frac{S-1}{S} (V_{m/1} - \psi) \right), \\ V_{m/1} &= -C_1 + \rho_1 V_2, \\ V_{g/1} &= -C_1 + \frac{\rho_1}{S} V_2, \text{ and} \\ V_{g/0} &= -C_0 + \rho_0 V_{g/1}. \end{aligned}$$

Note that in the second equation we used the fact that in the equilibrium of interest an established firm that created knowledge that does not match its capability will hold knowledge that it can sell for  $p - \psi$  and then it may purchase knowledge that matches its capability, getting  $V_{m/1} - p$ . Solving for  $V_2, V_{m/0}, V_{m/1}, V_{g/1}$  and  $V_{g/0}$  gives

$$V_2 = Y + \rho_2 \alpha_{m/0} \Phi_{m/0},$$

$$V_{m/0} = \alpha_{m/0} \Phi_{m/0}, \tag{49}$$

$$V_{m/1} = -C_1 + \rho_1 (Y + \rho_2 \alpha_{m/0} \Phi_{m/0}), \tag{50}$$

$$V_{g/1} = -C_1 + \frac{\rho_1}{S} (Y + \rho_2 \alpha_{m/0} \Phi_{m/0}) \text{ and} \tag{51}$$

$$V_{g/0} = -C_0 + \rho_0 \left( \frac{\rho_1}{S} Y + \frac{\rho_1 \rho_2 \alpha_{m/0} \Phi_{m/0}}{S} - C_1 \right), \tag{52}$$

where

$$\alpha_{m/0} = (S(1 - \rho_0\rho_1\rho_2))^{-1}$$

and

$$\Phi_{m/0} = S\rho_0(\rho_1Y - C_1) - SC_0 - (S - 1)\rho_0\psi.$$

Substituting equations (49) and (50) for  $V_{m/0}$  and  $V_{m/1}$  into equation (31) and solving for  $p$  give equation (35).

It remains to show that equation (32) is satisfied. Substituting into equation (32), equation (49) for  $V_{m/0}$ , equation (51) for  $V_{g/1}$ , and equation (35) for  $p$  give the necessary conditions (33) and (34). Q.E.D.

### Stationary Distribution of Firms' Population in Exploration Equilibrium

In exploration equilibrium, note that  $H_g = 0$  since a young firm prefers implementing its knowledge rather than selling it to an established firm. We can assume without loss of generality that  $H_m^0 = 1$ , that is all established firms that created unmatched knowledge will buy knowledge from another established firm, since in equilibrium they are exactly indifferent between buying a matched knowledge and incubating a new one. The market clearing condition for knowledge in exploration equilibrium becomes

$$\frac{\lambda_0(S-1)}{S}M_0H_m^0 + \lambda_2M_2H_m^2 = \frac{\lambda_0(S-1)}{S}M_0.$$

The left hand side is demand for knowledge, coming from established firms that created an unmatched knowledge (first term) and established firms finishing harvesting knowledge. The right hand side is supply of knowledge, coming from established firms that created an unmatched knowledge. The stationary equilibrium equations in the exploration case become

$$\begin{aligned} (\mu + \lambda_0)G_0 &= N \\ (\mu + \lambda_1)G_1 &= \lambda_0G_0 \\ (\mu + \lambda_1)M_1 &= \lambda_0M_0 \\ (\mu + \lambda_2)M_2 &= \frac{\lambda_1G_1}{S} + \lambda_1M_1 \\ (\mu + \lambda_0)M_0 &= \lambda_2M_2 \end{aligned}$$

Solving the system, we get

$$\begin{aligned}
G_0 &= \frac{1}{\mu + \lambda_0} N \\
G_1 &= \frac{\lambda_0}{(\mu + \lambda_0)(\mu + \lambda_1)} N \\
M_0 &= \frac{\lambda_0 \lambda_1 \lambda_2}{S(\mu + \lambda_0)[(\mu + \lambda_0)(\mu + \lambda_1)(\mu + \lambda_2) - \lambda_0 \lambda_1 \lambda_2]} N \\
M_1 &= \frac{\lambda_0}{\mu + \lambda_1} M_0 \\
M_2 &= \frac{\mu + \lambda_0}{\lambda_2} M_0.
\end{aligned}$$

The solution characterizes the distribution of firms in exploration equilibrium.

### Proof of Proposition 10

The idea behind the proof is that the firm that has experienced  $N$  previous failures to implement knowledge can replicate what the firm that has experienced  $N - 1$  previous failures with respect to the  $N - 1$  types that failed to implement in the past. Then the extra knowledge of the firm that has experienced  $N$  previous failures will ensure that it performs better than the firm that has experienced  $N - 1$  previous failures with respect to the knowledge type for which the firm with  $N - 1$  failures has no previous experience.

We first prove the following lemmas.

**Lemma 1** *Suppose that the assumptions stated in Proposition 8 hold. Then, one of the three cases is true. The first case is*

$$p - \psi + V_{g//0} [N] \geq V_{g//1} [N]$$

and therefore,

$$V_{g//0} [N] = -C_0 + \rho_0 (p - \psi + V_{g//0} [N]).$$

The second case is  $p - \psi + V_{g//0} [N] \leq V_{g//1} [N]$  and  $V_{g//1} [N] - p \leq V_{g//0} [N]$  and therefore,

$$V_{g//0} [N] = -C_0 + \rho_0 \frac{N}{S} (p - \psi + V_{g//0} [N]) + \rho_0 \frac{S - N}{S} V_{g//1} [N].$$

The third case is  $p - \psi + V_{g//0} [N] \leq V_{g//1} [N]$  and  $V_{g//0} [N] \leq V_{g//1} [N] - p$  and therefore,

$$V_{g//0} [N] = -C_0 + \rho_0 \frac{N}{S} (V_{g//1} [N] - \psi) + \rho_0 \frac{S - N}{S} V_{g//1} [N]$$

**Proof.** There are two maximum brackets in the right hand side of the equation

$$\begin{aligned} V_{g/0} [N] &= \rho_0 \frac{N}{S} \max \{p - \psi + V_{g/0} [N], V_{g/1} [N] - \psi\} \\ &\quad + \rho_0 \frac{S - N}{S} \max \{p - \psi + V_{g/0} [N], V_{g/1} [N]\} - C_0 \end{aligned}$$

Each bracket has two entries, so there are four cases depending on which entry is the maximum at each bracket. However,  $V_{g/1} [N] > V_{g/1} [N] - \psi$ , so it suffices to consider the three cases stated in the lemma. ■

**Lemma 2** *Suppose that the assumptions stated in Proposition 8 hold. Then,  $V_{g/i} [S - 1] > V_{g/i} [S - 2]$  for  $i = 0, 1$ .*

**Proof.** Note that

$$\begin{aligned} V_{g/1} [S - 2] &= -C_1 + \frac{1}{2} \rho_1 V_2 + \frac{1}{2} \rho_1 \max \{V_{g/0} [S - 1], V_{g/1} [S - 1] - p, 0\} \\ &= -C_1 + \frac{1}{2} \rho_1 V_2 + \frac{1}{2} \rho_1 \max \{V_{m/1} - p, 0\} \\ &= \frac{1}{2} V_{m/1} + \frac{1}{2} (-C_1 + \rho_1 \max \{V_{m/1} - p, 0\}) \\ &< V_{m/1} = V_{g/1} [S - 1] \end{aligned}$$

It remains to show that  $V_{g/0} [S - 2] < V_{g/0} [S - 1]$ . Using Lemma 1, it suffices to consider the following three cases. Suppose that (i)  $p - \psi + V_{g//0} [S - 2] \geq V_{g//1} [S - 2]$ . Then we get

$$V_{g//0} [S - 2] = -C_0 + \rho_0 (p - \psi + V_{g//0} [S - 2])$$

and

$$\begin{aligned}
V_{g/0}[S-1] &= -C_0 + \frac{1}{S}\rho_0 V_{g/1}[S-1] + \frac{S-1}{S}\rho_0 (p - \psi + \max\{V_{m/0}, V_{m/1} - p\}) \\
&\geq -C_0 + \frac{1}{S}\rho_0 V_{g/1}[S-1] + \frac{S-1}{S}\rho_0 (p - \psi + V_{g/0}[S-1]) \\
&\geq -C_0 + \frac{1}{S}\rho_0 (V_{g/0}[S-1] + p) + \frac{S-1}{S}\rho_0 (p - \psi + V_{g/0}[S-1]) \\
&> -C_0 + \frac{1}{S}\rho_0 (V_{g/0}[S-1] + p - \psi) + \frac{S-1}{S}\rho_0 (p - \psi + V_{g/0}[S-1]) \\
&= -C_0 + \rho_0 (p - \psi + V_{g//0}[S-1])
\end{aligned}$$

Therefore,

$$V_{g/0}[S-1] > \frac{-C_0 + \rho_0 (p - \psi)}{1 - \rho_0} = V_{g/0}[S-2]$$

Suppose now that (ii)  $p - \psi + V_{g//0}[S-2] \leq V_{g//1}[S-2]$  and  $V_{g//1}[S-2] - p \leq V_{g//0}[S-2]$ . Then

$$V_{g//0}[S-2] = -C_0 + \rho_0 \frac{S-2}{S} (p - \psi + V_{g//0}[S-2]) + \rho_0 \frac{2}{S} V_{g//1}[S-2],$$

which is equivalent to

$$\left(1 - \rho_0 \frac{S-2}{S}\right) V_{g//0}[S-2] = -C_0 + \rho_0 (p - \psi) + \rho_0 \frac{2}{S} V_{g//1}[S-2]$$

and

$$\begin{aligned}
V_{g/0}[S-1] &= -C_0 + \frac{S-2}{S}\rho_0 \max\{p - \psi + V_{g/0}[S-1], V_{g/1}[S-1] - \psi\} \\
&\quad + \frac{1}{S}\rho_0 \max\{p - \psi + V_{g/0}[S-1], V_{g/1}[S-1] - \psi\} \\
&\quad + \frac{1}{S}\rho_0 V_{g/1}[S-1] \\
&\geq -C_0 + \frac{S-2}{S}\rho_0 (p - \psi + V_{g/0}[S-1]) \\
&\quad + \frac{1}{S}\rho_0 (V_{g/1}[S-1] - \psi) + \frac{1}{S}\rho_0 V_{g/1}[S-1].
\end{aligned}$$

Therefore,

$$\begin{aligned}
\left(1 - \rho_0 \frac{S-2}{S}\right) V_{g/0}[S-1] &\geq -C_0 + \frac{S-2}{S}\rho_0 (p - \psi) + \frac{1}{S}\rho_0 (V_{g/1}[S-1] - \psi) \\
&\quad + \frac{1}{S}\rho_0 V_{g/1}[S-1].
\end{aligned}$$

It suffices to show that

$$\frac{1}{S}\rho_0 (V_{g/1} [S-1] - \psi) + \frac{1}{S}\rho_0 V_{g/1} [S-1] > \rho_0 \frac{2}{S} V_{g/1} [S-2].$$

Note that

$$\begin{aligned} \rho_0 \frac{2}{S} V_{g/1} [S-2] &= \rho_0 \frac{2}{S} \left\{ \frac{1}{2} V_{g/1} [S-1] + \frac{1}{2} (-C_1 + \max \{V_{g/1} [S-1] - p, 0\}) \right\} \\ &= \frac{1}{S} \rho_0 V_{g/1} [S-1] + \frac{1}{S} \rho_0 \{-C_1 + \max \{V_{g/1} [S-1] - p, 0\}\}. \end{aligned}$$

Since  $V_{g/1} [S-1] - \psi = p - \psi + V_{g/1} [S-1] - p > V_{g/1} [S-1] - p$ , the proof for case (ii) is complete. It remains to examine the case (iii)  $p - \psi + V_{g/0} [N] \leq V_{g/1} [N]$  and  $V_{g/0} [N] \leq V_{g/1} [N] - p$ . This follows exactly the same steps as case (ii), so we conclude that  $V_{g/0} [S-2] < V_{g/0} [S-1]$ . ■

**Lemma 3** *Suppose that the assumptions stated in Proposition 8 hold. Then,  $V_{g/1} [N] > V_{g/0} [N]$  for  $N = 0, 1, \dots, S-1$ .*

**Proof.**  $V_{g/1} [N] > V_{g/0} [N]$  holds for  $N = 0$  and  $N = S-1$  so it remains to show that it holds for  $N = 1, 2, \dots, S-2$ . When  $p - \psi + V_{g/0} [N] < V_{g/1} [N]$  the result is immediate since by assumption  $p - \psi \geq 0$ . It remains to show the result when  $p - \psi + V_{g/0} [N] \geq V_{g/1} [N]$ , so  $V_{g/0} [N] = -C_0 + \rho_0 (p - \psi + V_{g/0} [N])$ . Let  $N'$  be the highest  $N$  such that  $p - \psi + V_{g/0} [N] \geq V_{g/1} [N]$ , so for  $N > N'$ ,  $V_{g/1} [N] > p - \psi + V_{g/0} [N]$ . Note that

$$\begin{aligned} &V_{g/1} [N'] \\ &= -C_1 + \frac{1}{S-N'} \rho_1 V_2 + \frac{S-N'-1}{S-N'} \rho_1 \max \{V_{g/0} [N'+1], V_{g/1} [N'+1] - p, 0\} \\ &\leq p - \psi + V_{g/0} [N'] \end{aligned}$$

Then, since  $V_{g/1} [N] < p$ , we get

$$\begin{aligned} &\max \{V_{g/0} [N'], V_{g/1} [N'] - p, 0\} \\ &= \max \{V_{g/0} [N'], 0\} \\ &= \max \{-C_0 + \rho_0 (p - \psi + V_{g/0} [N']), 0\} \end{aligned}$$

Therefore,

$$\max \left\{ V_{g/0} [N' + 1], V_{g/1} [N' + 1] - p, 0 \right\} \geq \max \left\{ V_{g/0} [N'], V_{g/1} [N'] - p, 0 \right\}$$

and it is immediate that

$$V_{g//1} [N' - 1] < V_{g//1} [N'] \leq p - \psi + V_{g/0} [N']$$

By induction,  $V_{g//1} [0] < p - \psi + V_{g/0} [0]$ , which is a contradiction. ■

Now we are ready to prove Proposition 8. We will first show that  $V_{g//1} [N - 1] < V_{g//1} [N]$  for  $N = 0, 1, \dots, S - 1$ . Recall that

$$V_{g/1} [N] = -C_1 + \frac{1}{S - N} \rho_1 V_2 + \frac{S - N - 1}{S - N} \rho_1 \max \left\{ V_{g/0} [N + 1], V_{g/1} [N + 1] - p, 0 \right\}$$

From Lemma 2,  $V_{g//1} [N - 1] < V_{g//1} [N]$  holds for  $N = S - 1$ . We will show that  $V_{g//1} [S - 3] < V_{g//1} [S - 2]$ , that is the proposition holds for  $N = S - 2$ .

Note that

$$\begin{aligned} V_{g//1} [S - 3] &= -C_1 + \frac{1}{3} \rho_1 V_2 + \frac{2}{3} \rho_1 \max \left\{ V_{g/0} [S - 2], V_{g/1} [S - 2] - p, 0 \right\} \\ V_{g/1} [S - 2] &= -C_1 + \frac{1}{2} \rho_1 V_2 + \frac{1}{2} \rho_1 \max \left\{ V_{g/0} [S - 1], V_{g/1} [S - 1] - p, 0 \right\} \end{aligned}$$

Since  $\max \left\{ V_{g/0} [S - 1], V_{g/1} [S - 1] - p, 0 \right\} > \max \left\{ V_{g/0} [S - 2], V_{g/1} [S - 2] - p, 0 \right\}$ , we get

$$\begin{aligned} V_{g//1} [S - 3] &< -C_1 + \frac{1}{3} \rho_1 V_2 + \frac{2}{3} \rho_1 \max \left\{ V_{g/0} [S - 1], V_{g/1} [S - 1] - p, 0 \right\} \\ &= \frac{1}{3} V_{m/1} + \frac{2}{3} \left( -C_1 + \rho_1 \max \left\{ V_{g/0} [S - 1], V_{g/1} [S - 1] - p, 0 \right\} \right) \end{aligned}$$

Subtracting  $V_{g/1} [S - 2]$  from the last expression gives

$$V_{g/1} [S - 2] - V_{g//1} [S - 3] > \frac{1}{6} \left( V_{m/1} - C_1 + \rho_1 \max \left\{ V_{m/1} - p, 0 \right\} \right) > 0$$

Then by induction we get  $V_{g//1} [N - 1] < V_{g//1} [N]$  for  $N = 0, 1, \dots, S - 1$ .

It remains to show that  $V_{g//0} [N - 1] < V_{g//0} [N]$  for  $N = 0, 1, \dots, S - 1$ .

Recall that

$$\begin{aligned} V_{g/0} [N] &= \rho_0 \frac{N}{S} \max \left\{ p - \psi + V_{g/0} [N], V_{g/1} [N] - \psi \right\} \\ &\quad + \rho_0 \frac{S - N}{S} \max \left\{ p - \psi + V_{g/0} [N], V_{g/1} [N] \right\} - C_0 \end{aligned}$$

From Lemma 2,  $V_{g//0}[N-1] < V_{g//0}[N]$  holds for  $N = S-1$ . We will show that  $V_{g//0}[S-3] < V_{g//0}[S-2]$ , that is the proposition holds for  $N = S-2$ . Using Lemma 1, it suffices to consider three cases for each of  $V_{g//0}[S-3]$ ,  $V_{g//0}[S-2]$ . Suppose that  $p - \psi + V_{g//0}[S-2] \leq V_{g//1}[S-2]$  and  $V_{g//1}[S-2] - p \leq V_{g//0}[S-2]$ . Then

$$V_{g//0}[S-2] = -C_0 + \rho_0 \frac{S-2}{S} (p - \psi + V_{g//0}[S-2]) + \rho_0 \frac{2}{S} V_{g//1}[S-2]$$

Suppose that  $p - \psi + V_{g//0}[S-3] \leq V_{g//1}[S-3]$  and  $V_{g//1}[S-3] - p \leq V_{g//0}[S-3]$ . Then

$$V_{g//0}[S-3] = -C_0 + \rho_0 \frac{S-3}{S} (p - \psi + V_{g//0}[S-3]) + \rho_0 \frac{3}{S} V_{g//1}[S-3]$$

Following the same logic as in Lemma 2, we get

$$\begin{aligned} V_{g//0}[S-2] &= -C_0 + \rho_0 \frac{S-3}{S} (p - \psi + V_{g//0}[S-2]) \\ &\quad + \rho_0 \frac{1}{S} (p - \psi + V_{g//0}[S-2]) + \rho_0 \frac{2}{S} V_{g//1}[S-2] \end{aligned}$$

It suffices to show that

$$\rho_0 \frac{1}{S} (p - \psi + V_{g//0}[S-2]) + \rho_0 \frac{2}{S} V_{g//1}[S-2] > \rho_0 \frac{3}{S} V_{g//1}[S-3]$$

which holds, similarly to the derivation in Lemma 2. We can prove the other cases similarly, so  $V_{g//0}[S-3] < V_{g//0}[S-2]$  and by induction we get  $V_{g//0}[N-1] < V_{g//0}[N]$  for  $N = 0, 1, \dots, S-1$ . Q.E.D.

### Proof of Proposition 11

As preparation to prove the proposition 9, we introduce the following notation.

$$\mathbf{L} = \{N | V_{g/1}[N] - p + \psi < V_{g/0}[N]\},$$

$$\mathbf{D} = \{N | V_{g/1}[N] - p < V_{g/0}[N] \leq V_{g/1}[N] - p + \psi\}$$

and

$$\mathbf{B} = \{N | V_{g/0}[N] \leq V_{g/1}[N] - p\},$$

and prove the following two lemmas. Q.E.D.

**Lemma 4** *If  $N \in \mathbf{B}$  and  $N' > N$ , then  $N' \in \mathbf{B}$ .*

**Proof.** It suffices to show that if  $N \in \mathbf{B}$  then  $N + 1 \in \mathbf{B}$ . We prove the lemma by contradiction. Suppose that  $N + 1 \in \mathbf{D}$ . Then we have  $V_{g/0}[N] \leq V_{g/1}[N] - p$  and  $V_{g/1}[N + 1] - p < V_{g/0}[N + 1] \leq V_{g/1}[N + 1] - p + \psi$  so we get

$$V_{g/1}[N + 1] - V_{g/0}[N + 1] < p \leq V_{g/1}[N] - V_{g/0}[N] \quad (53)$$

Given that  $N + 1 \in \mathbf{D}$  and  $N \in \mathbf{B}$ , using equation (36), we have

$$\begin{aligned} V_{g/0}[N] &= -C_0 + \rho_0 \frac{N}{S} (V_{g/1}[N] - \psi) + \rho_0 \frac{S - N}{S} V_{g/1}[N] \\ V_{g/0}[N + 1] &= -C_0 + \rho_0 \frac{N + 1}{S} (p - \psi + V_{g/0}[N + 1]) \\ &\quad + \rho_0 \frac{S - N - 1}{S} V_{g/1}[N + 1] \end{aligned}$$

Subtracting these two equations and dividing both hand-sides by  $\rho_0$ , we get

$$\begin{aligned} &(V_{g/0}[N + 1] - V_{g/0}[N]) / \rho_0 \\ &= \frac{N + 1}{S} (p + V_{g/0}[N + 1] - V_{g/1}[N + 1]) + (V_{g/1}[N + 1] - V_{g/1}[N]) - \frac{1}{S} \psi \\ &< \frac{S - N - 1}{S} (V_{g/1}[N + 1] - V_{g/1}[N]) + \frac{N + 1}{S} (V_{g/0}[N + 1] - V_{g/0}[N]). \end{aligned}$$

The inequality follows because  $0 \leq \psi$  and  $p \leq V_{g/1}[N] - V_{g/0}[N]$  since  $N \in \mathbf{B}$ . After re-arranging this inequality, we have

$$\left(1 - \rho_0 \frac{N + 1}{S}\right) (V_{g/0}[N + 1] - V_{g/0}[N]) < \rho_0 \left(1 - \frac{N + 1}{S}\right) (V_{g/1}[N + 1] - V_{g/1}[N])$$

Since  $1 - \rho_0 \frac{N + 1}{S} > \rho_0 \left(1 - \frac{N + 1}{S}\right)$ , we get  $V_{g/0}[N + 1] - V_{g/0}[N] < V_{g/1}[N + 1] - V_{g/1}[N]$  which is contradiction to the equation (??). Therefore it cannot be the case that  $N + 1 \in \mathbf{D}$ . Similarly we can show that it cannot be the case that  $N + 1 \in \mathbf{L}$ . Therefore,  $N + 1 \in \mathbf{B}$  and the proof is complete. ■

**Lemma 5** *If  $N \in \mathbf{D}$ , then  $N + 1 \notin \mathbf{L}$ .*

**Proof.** Let  $N \in \mathbf{D}$ . Then, we have  $V_{g/1}[N] - p < V_{g/0}[N] \leq V_{g/1}[N] - p + \psi$ . Suppose that  $N + 1 \in \mathbf{L}$ . Then,  $V_{g/1}[N + 1] - p + \psi < V_{g/0}[N + 1]$ . Using

equation (36), we have

$$V_{g/0}[N] = -C_0 + \rho_0 \frac{N}{S} (p - \psi + V_{g/0}[N]) + \rho_0 \frac{S-N}{S} V_{g/1}[N]$$

and

$$V_{g/0}[N+1] = -C_0 + \rho_0 (p - \psi + V_{g/0}[N+1]).$$

Subtracting these two equations, we get

$$\begin{aligned} & V_{g/0}[N+1] - V_{g/0}[N] \\ = & \rho_0 (p - \psi + V_{g/0}[N+1] - V_{g/1}[N]) - \rho_0 \frac{N}{S} (p - \psi + V_{g/0}[N] - V_{g/1}[N]). \end{aligned}$$

After re-arranging, we have

$$(1 - \rho_0) (V_{g/0}[N+1] - V_{g/0}[N]) = \rho_0 \frac{S-N}{S} (p - \psi + V_{g/0}[N] - V_{g/1}[N]).$$

Note that the right hand side is non-positive. Then, since  $V_{g/0}[N+1] - V_{g/0}[N] \geq 0$ , we get  $V_{g/0}[N+1] = V_{g/0}[N]$ . Moreover, from  $N \in \mathbf{D}$  and  $N+1 \in \mathbf{L}$  we get  $V_{g/1}[N+1] - V_{g/0}[N+1] < p - \psi \leq V_{g/1}[N] - V_{g/0}[N] \implies V_{g/1}[N+1] < V_{g/1}[N]$  which is contradiction. Therefore it cannot be the case that  $N+1 \in \mathbf{L}$ . ■

Since  $0 \in \mathbf{D}$  by assumption, Lemma 4 and Lemma 5 imply that  $N \notin \mathbf{L}$  for  $\forall N$ . We now prove that  $V_{g/1}[N] - V_{g/0}[N] < V_{g/1}[N+1] - V_{g/0}[N+1]$  for two cases. One is  $N \in \mathbf{D}$  and the other is  $N \in \mathbf{B}$ . First, suppose that  $N \in \mathbf{D}$ . If  $N+1 \in \mathbf{B}$  the result is immediate. Suppose that  $N+1 \in \mathbf{D}$ . Then  $V_{g/1}[N] - p < V_{g/0}[N] \leq V_{g/1}[N] - p + \psi$  and  $V_{g/1}[N+1] - p < V_{g/0}[N+1] \leq V_{g/1}[N+1] - p + \psi$ . Then we have

$$\begin{aligned} V_{g/0}[N] &= -C_0 + \rho_0 \frac{N}{S} (p - \psi + V_{g/0}[N]) + \rho_0 \frac{S-N}{S} V_{g/1}[N] \\ V_{g/0}[N+1] &= -C_0 + \rho_0 \frac{N+1}{S} (p - \psi + V_{g/0}[N+1]) + \rho_0 \frac{S-N-1}{S} V_{g/1}[N+1] \end{aligned}$$

Subtracting the equations and re-arranging, we get

$$\begin{aligned} & V_{g/0}[N+1] - V_{g/0}[N] \\ < & \rho_0 \frac{S-N-1}{S} (V_{g/1}[N+1] - V_{g/1}[N]) + \rho_0 \frac{N+1}{S} (V_{g/0}[N+1] - V_{g/0}[N]) \end{aligned}$$

After re-arranging,

$$\begin{aligned} & \left(1 - \rho_0 \frac{N+1}{S}\right) (V_{g/0}[N+1] - V_{g/0}[N]) \\ & < \rho_0 \left(1 - \frac{N+1}{S}\right) (V_{g/1}[N+1] - V_{g/1}[N]) \end{aligned}$$

Since  $1 - \rho_0 \frac{N+1}{S} > \rho_0 \left(1 - \frac{N+1}{S}\right)$ , we get  $V_{g/0}[N+1] - V_{g/0}[N] < V_{g/1}[N+1] - V_{g/1}[N]$  or  $V_{g/1}[N] - V_{g/0}[N] < V_{g/1}[N+1] - V_{g/0}[N+1]$ . Suppose now that  $N \in \mathbf{B}$ . Then we have  $N+1 \in \mathbf{B}$ , so  $V_{g/0}[N] \leq V_{g/1}[N] - p$  and  $V_{g/0}[N+1] \leq V_{g/1}[N+1] - p$ . Then we have

$$V_{g/0}[N] = -C_0 + \rho_0 \frac{N}{S} (V_{g/1}[N] - \psi) + \rho_0 \frac{S-N}{S} V_{g/1}[N]$$

$$\begin{aligned} & V_{g/0}[N+1] \\ = & -C_0 + \rho_0 \frac{N+1}{S} (V_{g/1}[N+1] - \psi) + \rho_0 \frac{S-N-1}{S} V_{g/1}[N+1] \end{aligned}$$

Subtracting and re-arranging,

$$V_{g/0}[N+1] - V_{g/0}[N] < \rho_0 (V_{g/1}[N+1] - V_{g/1}[N]) - \rho_0 \frac{1}{S} \psi$$

That gives  $V_{g/0}[N+1] - V_{g/0}[N] < V_{g/1}[N+1] - V_{g/1}[N]$  or  $V_{g/1}[N] - V_{g/0}[N] < V_{g/1}[N+1] - V_{g/0}[N+1]$ . The proof is complete. Q.E.D.