Why do Reputable Agents Get Safer Projects? *

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Abstract

Safer firms receive funding from reputable venture capitalists and offer new securities underwritten by reputable investment banks. We offer a new explanation for these facts employing a moral-hazard model in which a firm and an agent are matched endogenously. If the agent is reputable, since she is more talented, her effort has a higher impact on output than otherwise. If the firm is safer, its output reflects the agent's hidden effort more accurately and, as a result, the agent's pay scheme tied with the output powerfully motivates her to exert effort. In equilibrium, a safer firm should be matched with a reputable agent since this combination allows to maximize effort of the reputable agent and minimizes deadweight costs associated with the moral hazard problem.

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Safer firms receive funding from reputable venture capitalists and offer new securities underwritten by reputable investment banks. We offer a new explanation for these facts employing a moral-hazard model in which a firm and an agent are matched endogenously. If the agent is reputable, since she is more talented, her effort has a higher impact on output than otherwise. If the firm is safer, its output reflects the agent's hidden effort more accurately and, as a result, the agent's pay scheme tied with the output powerfully motivates her to exert effort. In equilibrium, a safer firm should be matched with a reputable agent since this combination allows to maximize effort of the reputable agent and minimizes deadweight costs associated with the moral hazard problem.

1 Introduction

Fernando, Gatchev and Spindt (2005) find that more reputable investment bankers underwrite offerings by firms that have a lower risk of distress delisting. Sørensen (2006) finds that more experienced venture capitalists tend to invest in late-stage companies where risks associated with early ventures are already resolved. Existing two-sided matching models are not satisfactory for explaining these facts. For instance, Fernando, Gatchev and Spindt assume complementarity between firm's quality and underwriter's quality and derive that a better firm should be matched with a better underwriter. Although they use a variety of empirical proxies to measure quality of underwriter and issuer, their theoretical model is silent about what constitutes a "good firm" or a "good underwriter": Differently from Fernando, Gatchev and Spindt, Rosen (1982) explicitly models a "good firm" as a large firm. He argues that assigning an agent of superior talent to large firms is an efficient outcome, because greater talent filters through the entire firm by a recursive chain-of-command technology, thereby enhancing firm's productivity. Nevertheless, his model is silent about another important dimension of firm quality - firm risk, and its relation with agent's talent.[1]

To address this omission in the literature, we introduce two-sided matching into a standard principal-agent model developed by Holmström and Milgrom (1987). The economy is populated by many firms and many agents. Firms differ in riskiness of their businesses and agents differ in their productivity. These firms and agents are endogenously matched in a frictionless market. Once a firm-agent match is formed, the standard principal-agent story unfolds. The agent makes effort that affects the firm value. Because the agent's effort is not observable, the firm ties the agent's compensation with the observable firm value. Nevertheless, the firm value reflects not only the agent's effort but also uncontrollable risks. As a result, the agent's compensation becomes exposed to income risks. This exposure is costly because the agent is risk averse. To lessen the agent's risk exposure, the incentive provision

[1] A more reputable underwriter or VC may be matched with a safer firm because it is also a bigger firm. If so, the Rosen model is enough to explain the evidence cited above. Nevertheless, Fernando, Gatchev and Spindt (2005) find the aforementioned evidence, after controlling for firm size.

is compromised and the equilibrium effort level is inevitably lower than the first-best, i.e. the effort underprovision problem exists.

We solve for the equilibrium in which every principal-agent match is stable, i.e. simultaneous deviations of a principal and an agent from the equilibrium match do not make both simultaneously better off. Consistent with the aforementioned empirical evidence, our model predicts that a more productive agent should be matched with a safer firm in equilibrium. Intuition behind this result is as follows. A safer firm can offer high pay-performance sensitivity without making the agent's income very volatile. If a high pay-performance sensitivity is given, a more productive agent raises his or her effort level more than a less productive agent. As a consequence, when a safer firm and a more productive agent are matched, this combination powerfully lessens the effort underprovision problem. Because a safer firm and a more productive agent are complements one another, they should be matched in equilibrium.

A model without endogenous matching typically predicts that a riskier firm should on average pay more to its agents, in order to compensate for risks. Nevertheless, our model tells the opposite – one should observe a safer firm paying high compensation to its agents, resulting from endogenous matching. Agents working for a safer firm should be more productive in equilibrium and every firm is willing to pay more for such agents. As a consequence, a safer firm makes a higher compensation to retain its productive agents in equilibrium.

Our paper is closely related to Legros and Newman (2003), Serfes (2005) and Wright (2004), all of which study the assignment problem between a principal and an agent. They show that a more risk averse agent may end up with a riskier firm in equilibrium. Nevertheless, they are silent about the relation between the agent's productivity and the firm's risks. Our paper is also related to Lazear (1986), who argues that a more efficient worker chooses to work for performance pay, whereas a less efficient worker chooses to work for flat pay. In his model, the principal's choice of incentive schemes is exogenous. Our paper endogenizes this choice and associates it with the project characteristics. Namely, a safer firm offers a high-powered incentive scheme, whereas a riskier firm offers a pay scheme that is less sensitive to the agent's performance.

The paper is organized as follows. Section 2 introduces endogenous matching into the Holmström and Milgrom model. Section 3 derives predictions about the relation between agents' characteristics and principals' characteristics and about the agent's compensation. Section 4 concludes. Proofs of the propositions are gathered in the appendix.

2 Endogenous Matching in The Principal-Agent Model

In this section, we study the endogenous matching between a principal and an agent in the model developed by Holmström and Milgrom (1987); hereafter referred to as HM model. The authors developed a principal-agent model when the agent with CARA utility continuously controls the return process that follows the Brownian motion. One of HM model's results popular with empiricists is the linearity of the optimal incentive scheme with the closed-form solution for the pay-performance sensitivity.[2]

We begin with sketching a reduced-form version of HM model, and then proceed to study the market in which heterogeneous agents and heterogeneous principals are matched, given that, once the match is formed, the agent's compensation will be set according to HM model. In what follows, we address the principal as male and the agent as female.

[2]The results of the HM model are known to be robust. For instance, Sung (1995) found a similar result to the original HM model when the agent is allowed to control not only mean but also variance. In a static setting, Bittler, Moskowitz, and Vissing-Jørgenson (2005) also obtained qualitatively similar results to the HM model when the agent has CRRA utility.

2.1 Basic Setup

The agent's preference is the negative exponential utility defined on her pay, denoted by s minus disutility cost of providing effort e. The disutility cost c is a quadratic function of effort, $c = .5ke^2$, where k is a positive constant. As we will see in what follows, effort e is effectively equal to agent's input amount for production. Hence, rewriting the disutility cost function as $e = \sqrt{2c/k}$, we can see that k^{-1} is the measure of how effectively the agent can convert a given disutility cost c to her input e. In other words, k is an inverse measure of agent's productivity. Therefore, we call a low-k agent "productive".

Let ρ be the agent's coefficient of absolute risk aversion. Her utility function is then $v(s-c) = -\exp[-\rho(s-c)]$. The technology of the project that the principal possesses is stochastic and linear in e. To be concrete, $\widetilde{y} = e + \tilde{\varepsilon}$, where $\widetilde{y}$ is output and $\tilde{\varepsilon}$ is a nonobservable stochastic factor. We assume that $\widetilde{\varepsilon}$ is normally distributed with mean zero and variance σ^2.

As usual in other moral hazard models, the effort level e is privately known by the agent, whereas the realized output y is public information. The principal is risk neutral and rewards the agent based on y using the linear sharing rule $s = \alpha + \beta y$, where α is the fixed transfer from the principal to the agent, and β is a positive constant and a measure of pay-performance sensitivity. In this setting, the certainty equivalent payoff of the agent is

$$CE = \alpha + \beta e - .5ke^2 - .5\rho\beta^2\sigma^2. \tag{1}$$

If the agent chooses not to work for the principal, she would get a reservation payoff u that is expressed in certainty equivalent and determined in equilibrium. The principal's payoff is

$$\widetilde{\pi} = -\alpha + (1-\beta)\,\widetilde{y} = -\alpha + (1-\beta)\,(e + \tilde{\varepsilon}). \tag{2}$$

The principal chooses α, β, and e so as to maximize his expected profit $E\tilde{\pi}$, subject to the incentive compatibility condition of the agent $dCE/de = \beta - ke = 0$ and the participation constraint of the agent $CE \geqq u$.[3] As is well known, the solution for this problem is

$$\widehat{\alpha} = u - .5\widehat{\beta}^2 \left(k^{-1} - \rho\sigma^2\right), \tag{3}$$

$$\widehat{\beta} = \frac{1}{1 + k\rho\sigma^2}, \tag{4}$$

and

$$\widehat{e} = k^{-1}\widehat{\beta}. \tag{5}$$

The first-best of this model is achieved if effort is verifiable and we can drop the incentive compatibility condition from the optimization problem. The first-best effort e^* maximizes the expected output and therefore $e^* = k^{-1}$. According to equations (4) and (5), the second-best effort $\widehat{e}$ is always less than e^* – the effort underprovision problem.

Sticking $\widehat{\alpha}$, $\widehat{\beta}$, and $\widehat{e}$ into equation (2) and taking the expectation give the principal's expected profit

$$E\tilde{\pi} = \frac{.5}{k\left(1 + k\rho\sigma^2\right)} - u. \tag{6}$$

This equation implies that the principal's expected profit is decreasing in σ^2. Intuitions behind the negative dependence of the expected profit on σ^2 are in the informativeness criterion proposed by Holmström (1979). According to the informativeness criterion, the efficiency of the agency model is reduced when the agent is rewarded based on the performance metric that reflects agent's effort level less accurately than other metrics.[4] Due to the

[3]Alternatively, we can formulate the problem such that the agent, instead of the principal, has all the bargaining power and sets the contract to maximize her certainty equivalent payoff, subject to that the principal gets his reservation payoff. Under this alternative specification, we still obtain the same pay-performance sensitivity as the optimum. This is because we can break down the contracting problem into two. One is to maximize the joint output and the other is to divide the joint output between the principal and the agent. The former is achieved by setting β appropriately and the latter is achieved by setting α appropriately. There is no conflict of interest between the principal and the agent for the former problem, and therefore no matter which party designs the contract, the resulting pay-performance sensitivity will be the same.

[4]Kim (1995) goes a step further beyond the Holmström's informativeness criterion and shows that the performance metric can be ordered by mean-preserving spreads of the likelihood function. A likelihood

stochastic production function, the accuracy of y as the measure of agent's effort e decreases when σ^2 increases, and therefore the principal-agent contract becomes less efficient and the principal's expected profit decreases. The equation (6) also implies that, other things being equal, including u, a principal becomes better off if a more efficient and/or less risk averse agent is hired. Thus, every principal is naturally willing to pay more for such an agent, and therefore in equilibrium the agent's outside option, u, should be positively related to her efficiency and negatively related to her risk aversion.

The linear sharing rule implies that agent's expected compensation can be written as $w = \alpha + \beta e$. Then, evaluating the endogenous variables at the optimum (i.e., setting $\alpha = \widehat{\alpha}, \beta = \widehat{\beta}$, and $e = \widehat{e}$), gives the expected output

$$Ey = \widehat{e} = \frac{1}{k\left(1 + k\rho\sigma^2\right)} \tag{7}$$

and the expected agent's compensation

$$w = u + .5\left(k\widehat{e}^2 + \rho\widehat{\alpha}_1^2\sigma^2\right) = u + .5Ey. \tag{8}$$

2.2 Endogenous Matching

We now describe the economy in which many agents and many principals are populated, and ask the question of which agent should be matched with which principal in equilibrium. We characterize a principal by attributes of the project that he possesses. As a consequence, assigning an agent to a principal is equivalent to assigning an agent to the principal's project. To make the exposition that follows clearer, we often speak in terms of assignment of agents to projects, instead to the principals that possess them.

function that is created by mean-preserving spread of another likelihood function, is associated with a higher efficiency of the agency model than the original likelihood function. One can understand the intuition behind this result by recalling that the optimal compensation schedule is increasing in the likelihood ratio of the signal relative to the induced effort level. When the likelihood function is flatter (i.e. mean-preserving spread), then the agent's compensation becomes less volatile and therefore the inefficiency of the agency problem associated with sub-optimal risk sharing is reduced.

For simplicity, we assume that the numbers of agents and projects are both equal to N. The population of agents is expressed by $M = \{m_1, m_2, ..., m_N\}$ and the population of projects is expressed by $F = \{f_1, f_2, ..., f_N\}$. Agent i, m_i, is characterized by $\{k_i, \rho_i\}$ and project j, f_j, is characterized by σ_j^2. Our equilibrium concept is the core. Specifically, an equilibrium consists of a matching correspondence $\mathfrak{M}^* : F \rightrightarrows M$ that specifies the type of agent to which each project is matched, and a payoff allocation $u^* : M \to \mathbb{R}$ and $E\tilde{\pi}^* : F \to \mathbb{R}$ specifying the equilibrium utility achieved by each party. The key property of the equilibrium is the stability condition: if the set of $\mathfrak{M}$, u, and $E\tilde{\pi}$ is in equilibrium, then there should be no pair of an agent and a principal such that they are not matched, but both of them can achieve higher payoffs if they were matched.

Conveniently, in our model, the optimal sharing rule $\widehat{\beta}$ and the optimal effort $\widehat{e}$ are independent of agents and principals reservation payoffs that determine how surplus is split up. Therefore, in order to solve for equilibrium, we can treat our model as if utility were transferable between an agent and a principal.[5] As a consequence, we can characterize the matching patterns in equilibrium by examining cross derivatives of the joint payoff function

$$\Phi \equiv E\tilde{\pi} + u = \frac{.5}{k\left(1 + k\rho\sigma^2\right)}.$$

Before we proceed to characterize the matching equilibrium, it is useful to characterize the joint payoff function Φ. This function is decreasing in k, ρ, and σ^2. Therefore, in terms of the contribution to joint output, a project with low σ^2 is superior to one with high σ^2, and an agent with low k and/or low ρ is superior to an agent with high k and/or high ρ. Intuition behind these rankings is straightforward. When σ^2 is low, the agent engaged in such a project is exposed to less uncontrollable risk and therefore more effort can be induced by a high pay-performance sensitivity. Therefore, Φ is decreasing in σ^2, that is, a safer project is desirable. When k is low, the agent makes more effort given the same disutility of effort. As a result, Φ is decreasing in k, that is, a more efficient agent is

[5] See Li and Ueda (2004) for the formal derivation of this point.

desirable. When ρ is low, the principal can offer high pay-performance sensitivity without hurting the agent's expected utility too much and therefore can induce higher effort. This results in a higher joint output, and therefore a less risk averse agent is desirable.

3 Results

In this section, we characterize equilibrium of the endogenous matching model described in the previous section and its implication on agents' compensation. First, we study the equilibrium relation between agents' characteristics and principals' characteristics. Second, we study the distribution of agents' compensation.

3.1 Sorting in Project Risks and Agent's Productivity

We are now going to claim the main proposition of this paper.

Proposition 1 *Other things being equal, an agent with lower k should be matched with a project with lower σ^2.*

Intuitions behind this proposition are as follows. As the equation (5) shows, a more efficient agent is more responsive to high pay-performance sensitivity, meaning that she increases her effort by a large degree when the pay-performance sensitivity is raised. For the purpose of lessening the effort under-provision problem, this responsiveness is better exploited with a safer project, in that it can offer a high pay-performance sensitivity without raising the agent's risk exposure too much.

This proposition is consistent with the previous findings that a safer firm tends to raise funds from reputable venture capitalists and issues new shares underwritten by reputable investment bankers. Both reputable venture capitalists and reputable investment bankers are presumably more productive than others.[6]

Further, both, venture capitalist - firm and investment bank - firm, relationships appear to satisfy our underlying assumptions of moral hazard. A venture capitalist provides not only funds but also value-enhancing service to its portfolio firms. A moral hazard problem concerning the venture capitalist's effort is likely to be present, as such effort is largely unobservable. An important function of underwriters is to maintain the price of newly issued securities. This function involves complex tasks such as managing the expectation of the markets and trading the newly issued securities at the right timing. As a consequence, it is difficult to observe the underwriter's effort itself, and a moral hazard problem exists.

Our result offers a new explanation with regards to an interesting finding about the matching between crops and farmers. Ackerberg and Botticini (2002) study the relationship between farmers' wealth and crop types in early Renaissance Tuscany. Their sample include cereal and vine farmers. Cereal crops are considered to reflect farmers' effort more accurately than vines, as cereals are less subject to weather than vines. Ackerberg and Botticini find that wealthier farmers were more likely to cultivate cereals instead of vines. Farmers' wealth may reflect their productivity because productive farmers tend to accumulate wealth. With this interpretation, the tendency of wealthy farmers to cultivate cereals is consistent with our result that a productive agent tends to undertake a safer project.

3.2 Project Risks and Agent's Compensation

What does the model imply regarding the agent's compensation level w and compensation scheme β? To examine this question, we begin with discussing agent's payoff u. We can characterize agents' payoffs by examining difference of the joint payoff function. To see this point, we introduce an additional notation. Let $m(j)$ be the equilibrium matching function

[6] In particular, Sorensen uses experience to identify reputable VC, and Fernando et al. employ relative market share of an underwriter and relative position of an underwriter on tombstone announcement to proxy underwriter reputation. More productive agents tend to survive in their business, accumulate expertise, and establish their advantageous position in the market.

such that if $m(j) = i$, then the match of f_j and m_i is in the equilibrium. Also, let the joint payoff function be

$$\Phi(i,j) \equiv \frac{.5}{k_i\left(1 + k_i\rho_i\sigma_j^2\right)} \quad \forall i,j. \tag{9}$$

If the principal j' attempts to deviate from the equilibrium match and to be matched with $i' \neq m(j')$, then he has to give the agent i' at least $u_{i'}^*$. The stability condition implies that such a deviation is not profitable for the principal. Therefore,

$$\Phi\left(m(j'), j'\right) - u_{m(j')}^* \geq \Phi\left(i', j'\right) - u_{i'}^*, \quad \forall i' \neq m(j'). \tag{10}$$

If we focus on two principals, $f_{j'}$ and $f_{j''}$, then we can combine equation (10) for these two principals as

$$\Phi\left(m(j''), j''\right) - \Phi\left(m(j'), j''\right) \geq u_{m(j'')}^* - u_{m(j')}^* \geq \Phi\left(m(j''), j'\right) - \Phi\left(m(j'), j'\right). \tag{11}$$

As a result, the difference in the joint payoffs defines the upper bound and lower bound of the difference in the agents' payoffs. This property allows us to characterize the distribution of agents' payoffs. For instance, if $\Phi(m(j''), j') - \Phi(m(j'), j')$ is positive, one can conclude that $u_{m(j'')}^* > u_{m(j')}^*$.

Suppose that $\sigma_{j'}^2 \geq \sigma_{j''}^2$. Then, does $m(j'')$ receive a higher payoff than $m(j')$? The answer is yes if both principals prefer $m(j'')$ to $m(j')$, meaning that $\Phi(m(j''), j'') > \Phi(m(j'), j'')$ and $\Phi(m(j''), j') > \Phi(m(j'), j')$. Under these conditions, equation (11) implies that $u_{m(j'')}^* > u_{m(j')}^*$. Nevertheless, for instance, if $f_{j'}$ prefers $m(j')$ to $m(j'')$ (i.e., $\Phi[m(j''), j'] < \Phi[m(j'), j']$), there is no guarantee that $u_{m(j'')}^* > u_{m(j')}^*$. Therefore, to predict which agent gets a higher payoff than others, we restrict ourselves into the economy in which principals unanimously agree on the ranking of agents. Further, to characterize the distribution of w instead of u, we also have to add the costs of effort and risk bearing to u^*, as in equation (8).

We now state our first result on agents' compensation.

Proposition 2 *If ρ is constant for all agents, then $u_{m(j')} \leq u_{m(j'')}$, $w_{m(j')} \leq w_{m(j'')}$ and $\beta_{m(j')} \leq \beta_{m(j'')}$ for $\sigma^2_{j'} \geq \sigma^2_{j''}$.*

If ρ is constant for all agents, principals rank agents based only on k and their rankings should all agree. As every principal competes for a low-k agent, such an agent should receive a high payoff u in equilibrium. This observation, combined with Proposition 1 that predicts the positive correlation of k and σ^2 in the matching equilibrium, implies that an agent matched with a low σ^2 project should receive a higher payoff u. The combination of a low-k agent and a low-σ^2 project should lead to a high pay-performance sensitivity due to equation (4) and to a high effort. As a consequence, a low-k agent receives a high compensation w not only because she is desirable but also because her high effort requires high reward.

If ρ varies across agents and principals rank agents based not only on k but also on ρ, the characteristics of agent's compensation depends on the joint distribution of k and ρ in the agent's population. To formalize this point, we introduce the following assumptions.

Assumption 1 *If $k_i < k_j$, then $\rho_i \leq \rho_j$ for $\forall i \neq j$.*

This assumption ensures that more efficient agents are at least as risk tolerant as less efficient agents. Therefore, if this assumption is met, more efficient agents are also at least as desirable as less efficient agents in the risk aversion dimension. As a consequence, all principals agree on the ranking of agents since the joint payoff Φ is decreasing in both k and ρ for any σ^2. How plausible is this assumption? If we interpret the agent's CARA utility as a local approximation to CRRA utility, the risk aversion coefficient ρ must be negatively related with the agent's wealth level. Since a more efficient agent can earn more than a less efficient agent, a more efficient agent is likely wealthier and therefore at least as risk tolerant as a less efficient agent. Therefore, we believe that this assumption makes some sense.[7]

Assumption 2 *For any combination of agent and project,* $k\rho\sigma^2 > 1$.

This assumption implies that for any arbitrary principal-agent match the optimal pay-performance sensitivity is less than 0.5. (Note that the optimal pay-performance sensitivity is equal to $\left(1+k\rho\sigma^2\right)^{-1}$.) Further, Assumption 2 implies the following proposition.

Proposition 3 *Other things being equal, if Assumption 2 is met, then an agent with lower* ρ *should be matched with a project with lower* σ^2*.*

Intuition behind this result is not as straightforward as for the other results. To understand the intuition, it is convenient to divide the effect of an increase in σ^2 on welfare into two effects: the portfolio effect and the incentive effect. The portfolio effect concerns the risk sharing between the principal and the agent, and is a direct effect of σ^2 on the agent's utility with $\widehat{\beta}$ being given. According to the last term of equation (1), the agent's utility decreases in σ^2 and this reduction of the agent's utility is severer if ρ is higher. Therefore, according to the portfolio effect, a more risk averse agent should be willing to pay more to avoid high σ^2 than a less risk averse agent. Note that this portfolio effect is bigger when $\widehat{\beta}$ is greater. There is also an indirect effect: an increase in σ^2 reduces $\widehat{\beta}$ and affects the agent's incentive (the incentive effect). Unlike the portfolio effect, the incentive effect concerns the agent's effort and thereby the level of output. Equation (4) suggests that the impact of σ^2 on $\widehat{\beta}$ depends on ρ in two ways. Differentiating $\widehat{\beta}$ with respect to σ^2 gives

$$\frac{\partial\widehat{\beta}}{\partial\sigma^2} = -\left(k\rho \times \widehat{\beta}^2\right).$$

According to this derivative, first, high ρ *increases* the negative impact of σ^2 on incentives

[7] For example, using the data on CEO, Becker (2006) finds that wealth and efficiency are not significantly correlated. This no-correlation also satisfies Assumption 1.

through the first term $k\rho$, and second, high ρ *decreases* the same negative impact through the second term $\widehat{\beta}^2$ that is decreasing in ρ. Similarly to the portfolio effect, the power of the first term linearly increases in $\widehat{\beta}^2$. Therefore, when $\widehat{\beta}$ is small enough, both the portfolio effect and the first part of the incentive effect become small, and as a consequence the second part of the incentive effect outweighs - higher ρ weakens the negative impact of σ^2 on the incentive $\widehat{\beta}$. Hence, a more risk averse agent should be willing to bid more for a riskier project than a less risk averse agent should be.

The following proposition states that Assumption 1 and Assumption 2 are sufficient for higher compensation at safer projects.

Proposition 4 *If Assumption 1 and Assumption 2 both hold, then,* $u_{m(j')} \leq u_{m(j'')}$, $w_{m(j')} \leq w_{m(j'')}$ *and* $\beta_{m(j')} \leq \beta_{m(j'')}$ *for* $\sigma^2_{j'} \geq \sigma^2_{j''}$.

If Assumption 1 holds, every principal competes for a low-k agent and such an agent should thereby receive a high payoff u in equilibrium. Assumption 2 guarantees that a low-k agent should be matched with a low-σ^2 project even though a low-k agent is less risk averse. Similarly to the intuition behind Proposition 2, an agent at a safer project gets high payoff, high compensation, and pay-performance sensitivity.

4 Concluding Remarks

When principals and agents choose each other from the pool of heterogeneous agents and the pool of heterogeneous principals, respectively, the economics of endogenous matching suggests that a systematic relation may exist between principals' characteristics and agents' characteristics. To explore the implication of endogenous matching in the presence of moral hazard, we extend the popular version of the principal-agent model developed by Holmström and Milgrom (1987), and ask which agent should be matched with which project, given that the agent's compensation scheme will be set according to the model by Holmström and Milgrom. We find that a more efficient agent should be matched with a safer project, as this combination greatly reduces the effort underprovision problem. This result explains why more reputable investment banks underwrite safer firms and why more experienced venture capitalists tend to fund later-stage firms where uncertainty is reduced relative to early-stage firms.

Appendix

Proof of Proposition 1. The proof of this proposition is done by taking cross derivatives of Φ. Observe that

$$\frac{\partial^2 \Phi}{\partial k \partial \left(\sigma^2\right)} = \frac{\rho^2 \sigma^2}{\left(1 + k\rho\sigma^2\right)^3} > 0.$$

This completes the proof. ■

Proof of Proposition 2. Let $k(j)$ be k of $m(j)$ for any j. Due to Proposition 1, $k(j'') \leq k(j')$. By equations (9) and (10), it is immediate that $u_{m(j')} \leq u_{m(j'')}$. By equation (4), $\beta_{m(j')} \leq \beta_{m(j'')}$. Similarly, Ey is not less for $f_{j''}$ than for $f_{j'}$, due to equation (7). As both u and Ey are higher for $m_{j''}$, by equation (8), $w_{m(j'')} \geq w_{m(j')}$. ■

Proof of Proposition 3. Similarly to the proof of Proposition 1, observe that

$$\frac{\partial \Phi}{\partial \rho \partial \left(\sigma^2\right)} = \frac{k\rho\sigma^2 - 1}{2\left(1 + k\rho\sigma^2\right)^3}.$$

This completes the proof. ■

Proof of Proposition 3. Let $k(j)$ and $\rho(j)$ be k and ρ of $m(j)$ for any j, that is, $k(j)$ and $\rho(j)$ are the characteristics of the agent that is matched with firm j in equilibrium. We first demonstrate that $k(j'') \leq k(j')$ and $\rho(j'') \leq \rho(j')$, and proceed to prove that these results imply $u_{m(j')} \leq u_{m(j'')}$, $\beta_{m(j')} \leq \beta_{m(j'')}$, and also $w_{m(j')} \leq w_{m(j'')}$.

Pick two arbitrary agents $m_{i'}$ and $m_{i''}$ such that $k_{i'} \geq k_{i''}$. Then, by assumption $\rho_{i'} \geq \rho_{i''}$. To demonstrate, $k(j'') \leq k(j')$ and $\rho(j'') \leq \rho(j')$, it suffices to show that

$$\Phi\left(i'', j''\right) - \Phi\left(i', j''\right) \geq \Phi\left(i'', j'\right) - \Phi\left(i', j'\right), \tag{12}$$

and this is what we are going to do now. Let $A(i) = k_i \rho_i$ and $B(j) = \sigma_j^2$. Using the definition of Φ in equation (9) and subtracting the right hand side from the left hand side of equation (12) give

$$\begin{aligned}
& \left(\Phi\left(i'',j''\right)-\Phi\left(i',j''\right)\right)-\left(\Phi\left(i'',j'\right)-\Phi\left(i',j'\right)\right) \\
= \; & .5k_{i''}^{-1}\left(\left(1+A\left(i''\right)B\left(j''\right)\right)^{-1}-\left(1+A\left(i''\right)B\left(j'\right)\right)^{-1}\right) \\
& -.5k_{i'}^{-1}\left(\left(1+A\left(i'\right)B\left(j''\right)\right)^{-1}-\left(1+A\left(i'\right)B\left(j'\right)\right)^{-1}\right) \\
\geq \; & .5k_{i'}^{-1}\left(\left(1+A\left(i''\right)B\left(j''\right)\right)^{-1}-\left(1+A\left(i''\right)B\left(j'\right)\right)^{-1}\right) \\
& -.5k_{i'}^{-1}\left(\left(1+A\left(i'\right)B\left(j''\right)\right)^{-1}-\left(1+A\left(i'\right)B\left(j'\right)\right)^{-1}\right). \qquad (13)
\end{aligned}$$

The inequality follows as the inside of the large bracket in the second line is positive. Accordingly, to prove equation (12), it remains to demonstrate that the function $\Upsilon \equiv (1+A(i)B(j''))^{-1}-(1+A(i)B(j'))^{-1}$ is decreasing in $A(i)$, since $A(i') \geq A(i'')$. Differentiating Υ with respect to $A(i)=A$ gives

$$\begin{aligned}
\frac{d\Upsilon}{dA} &= -\left(1+A(i)B\left(j''\right)\right)^{-2}B\left(j''\right)+\left(1+A(i)B\left(j'\right)\right)^{-2}B\left(j'\right) \\
&= \frac{B(j')-B(j'')}{\left(1+A(i)B(j')\right)^{2}\left(1+A(i)B(j'')\right)^{2}}.
\end{aligned}$$

Since $B(j') \leq B(j'')$, $d\Upsilon/dA \leq 0$ and, as a result, equation (12) is proved and this establishes that $k(j'') \leq k(j')$ and $\rho(j'') \leq \rho(j')$.

We now proceed to prove that these results imply $u_{m(j')} \leq u_{m(j'')}$, $\beta_{m(j')} \leq \beta_{m(j'')}$ and also $w_{m(j')} \leq w_{m(j'')}$. By equations (9) and (10), it is immediate that $u_{m(j')} \leq u_{m(j'')}$. Note that $\sigma_{j''} \leq \sigma_{j'}$, $k(j'') \leq k(j')$ and $\rho(j'') \leq \rho(j')$. As a result, $\beta_{m(j')} \leq \beta_{m(j'')}$ by equation (4) and Ey is not less for $f_{j''}$ than $f_{j'}$, due to equation (7). As both u and Ey are higher for $m_{j''}$, by equation (8), $w_{m(j'')} \geq w_{m(j')}$. ■

References

[1] Ackerberg, Daniel A., and Maristella Botticini, 2002, "Endogenous Matching and the Empirical Determinants of Contract Form," *Journal of Political Economy* 110(3), 564-591.

[2] Becker, Bo, 2006, "Wealth and Executive Compensation, " *Journal of Finance* 61(1), 379-397.

[3] Bitler, Marianne P., Tobias J. Moskowitz, and Annette Vissing-Jørgensen, 2005, "Testing Agency Theory with Entrepreneur Effort and Wealth," *Journal of Finance* 60(2), 539-576.

[4] Fernando, Chitru, Vladimir Gatchev, and Paul Spindt, 2005, "Wanna Dance? How Firms and Underwriters Choose Each Other," *Journal of Finance,* forthcoming.

[5] Holmström, Bengt, 1979, "Moral Hazard and Observability," *Bell Journal of Economics* 10, 74-91.

[6] Holmström, B. and P. Milgrom, 1987, "Aggeregation and Linearity in Principal-Agent Contract", *Econometrica* 55(2), 303-328.

[7] Kim, Son Ku, 1995, "Efficiency of an Information System in an Agency Model," *Econometrica* 63(1), 89-102.

[8] Lazear, Edward P., 1986, "Salaries and Piece Rates," *Journal of Business* 59(3), 405-431.

[9] Legros, Patrick, and Andrew F. Newman, 2003, "Beauty Is a Beast, Frog Is a Prince: Assortative Matching with Nontransferabilities," mimeo.

[10] Li, Fei, and Masako Ueda, 2004, "CEO-Firm Match and Principal-Agent Theory," CEPR Working Paper.

[11] Rosen, Sherwin, 1982, "Authority, control, and the distribution of earnings," *Bell Journal of Economics* 13, 311-323.

[12] Serfes, K., 2005, "Risk sharing vs. incentives: Contract design under two-sided heterogeneity" *Economics Letters* 88(3), 343-349.

[13] Sørensen, Morten, 2006, "How Smart Is Smart Money? An Empirical Two-Sided Matching Model of Venture Capital," *Journal of Finance*, forthcoming.

[14] Sung, Jaeyong, 1995, "Linearity with project selection and controllable diffusion rate in continuous-time principal-agent problems," *Rand Journal of Economics* 26 (4), 720-743.

[15] Wright, D.J., 2004, The Risk and Incentives Trade-off in the Presence of Heterogeneous Managers, *Journal of Economics* 83, December, 209-223.